Logic, Complexity, and Games

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Abstract

In this talk, I will discuss an approach to the P = NP question via the correspondence between logic and complexity. The main focus will be on the possible use of Ehrenfeucht-Fraïssé games.

1. Summary of Talk

The *computational complexity* of a problem is the amount of resources, such as time or space, required by a machine that solves the problem. Complexity theory traditionally has focused on the computational complexity of problems. A more recent branch of complexity theory focuses on the *descriptive complexity* of problems, which is the complexity of describing problems in some logical formalism [4]. One of the exciting developments in complexity theory is the discovery of a very intimate connection between computational and descriptive complexity. In particular, the author showed [2] that the complexity class NP coincides with the class of properties of finite structures expressible in existential second-order logic, otherwise known as Σ^1_1. Because of this connection, a potential method of proving lower bounds in complexity theory is to prove inexpressibility results in the corresponding logic.

In this talk, the author will focus on our attempts to resolve the P = NP question by this approach. To prove P $\neq$ NP, it is sufficient to prove NP $\neq$ co-NP. By the equivalence of NP and Σ^1_1, it follows that NP $\neq$ co-NP if and only if $\Sigma^1_1 \neq \Pi^1_1$ (where Π^1_1 is the complement of Σ^1_1).

To try to prove that $\Sigma^1_1 \neq \Pi^1_1$, we have the tool of Ehrenfeucht-Fraïssé games [1, 3]. In the talk, we discuss some of the successes of Ehrenfeucht-Fraïssé games in proving lower bounds, and some of the tools that have been developed to facilitate the use of these games. We also discuss the stumbling blocks we have run into.

References

[1] A. Ehrenfeucht. An application of games to the completeness problem for formalized theories. *Fund. Math.*, 49:129–141, 1961.

[2] R. Fagin. Generalized first–order spectra and polynomial–time recognizable sets. In R. M. Karp, editor, *Complexity of Computation, SIAM-AMS Proceedings, Vol. 7*, pages 43–73, 1974.

[3] R. Fraïssé. Sur quelques classifications des systèmes de relations. *Publ. Sci. Univ. Alger. Sér. A*, 1:35–182, 1954.

[4] N. Immerman. Descriptive and computational complexity. In J. Hartmanis, editor, *Computational Complexity Theory, Proc. Symp. Applied Math., Vol. 38*, pages 75–91. American Mathematical Society, 1989.