Interactive Bayesian Probabilistic Programming and Debugging

IBM Programming Languages Day
December 4, 2017

Javier Burroni, Arjun Guha, David Jensen

Definition of (Bayesian) Probabilistic Programming Languages

Regular PL with two special constructs:

- `sample`
- `observe`

Gordon, A. D., Henzinger, T. A., Nori, A. V. & Rajamani, S. K. Probabilistic programming. in Proceedings of the on Future of Software Engineering 167–181 (ACM, 2014).

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```
random Real Mu() ~ Gaussian(100.0, 10.0);
random Real X() ~ Gaussian(Mu(), 15.0);

obs X() > 120;
query Mu() > 100;
```

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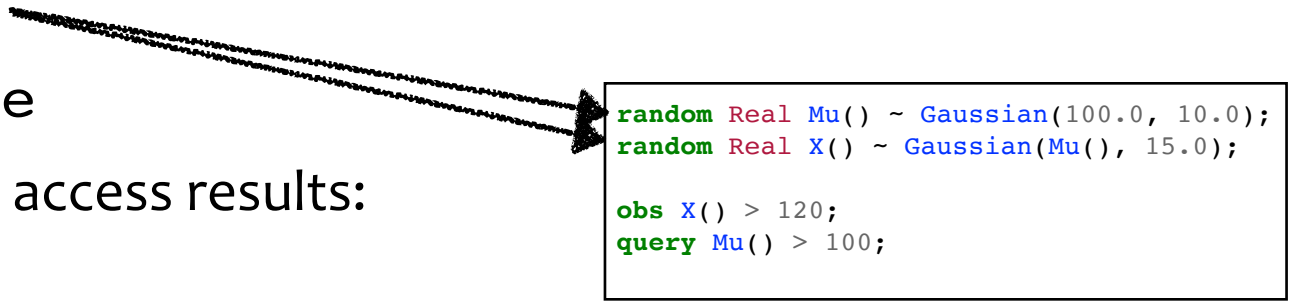

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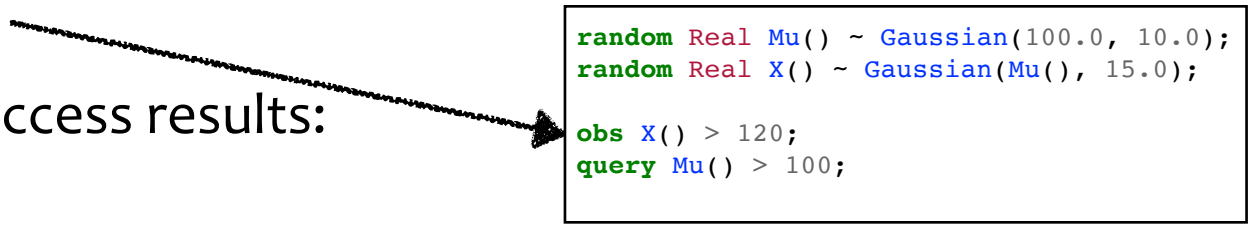

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Example of PPL

```
 0: type City;
 1: type PrepLevel;
 2: type DamageLevel;

 3: random City First ~ UniformChoice({c for City c});
 4: random City NotFirst ~ UniformChoice({c for City c: c != First});
 5: random PrepLevel Prep(City c) ~
 6:   if (First == c) then Categorical({High -> 0.5, Low -> 0.5})
 7:   else case Damage(First) in
 8:             {Severe -> Categorical({High -> 0.9, Low -> 0.1}),
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10: random DamageLevel Damage(City c) ~
11:    case Prep(c) in {High -> Categorical({Severe -> 0.2, Mild -> 0.8}),
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13: distinct City A, B;
14: distinct PrepLevel Low, High;
15: distinct DamageLevel Severe, Mild;

16: obs Damage(First) = Severe;

17: query Damage(NotFirst);
```

Milch, B. et al. BLOG: Probabilistic models with unknown objects. in (2005).

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13: distinct City A, B;
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```

```
query Damage(NotFirst)

Mild      0.733241
Severe    0.266759
dtype: float64
```

Milch, B. et al. BLOG: Probabilistic models with unknown objects. in (2005).

Interactive limitations

- New queries require re-execution of the entire program
  - ✦ Not efficient as neither data nor generative model were changed

Our approach

- Perform backward inference only once
- The result of inference is a posterior distribution over **traces** — FOL structures.
- New keyword: `inspect(expr) -> value`. Evaluate the expression in a trace
- `query(expr)` implemented as application of `inspect` over a sample of the posterior distribution of **traces**.

Advantages

- No need to know the queries before running inference
  - ✦ Allows interactively querying of the posterior distribution

Dynamically querying

```
 0: type City;
 1: type PrepLevel;
 2: type DamageLevel;

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13: distinct City A, B;
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15: distinct DamageLevel Severe, Mild;

16: obs Damage(First) = Severe;

17: query Damage(NotFirst);
```

```
query Damage(A)

Severe    0.641092
Mild      0.358908
```

```
query if Damage(A) == Severe then Prep(A) else Prep(B);
Low     0.741546
High    0.258454
```

Advantages

- No need to know the queries before running inference
  - ✦ Allows interactively querying of the posterior distribution
- `inspect(expr)` accepts any valid BLOG expression

Inspect one world

```
 0: type City;
 1: type PrepLevel;
 2: type DamageLevel;

 3: random City First ~ UniformChoice({c for City c});
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15: distinct DamageLevel Severe, Mild;

16: obs Damage(First) = Severe;

17: query Damage(NotFirst);
```

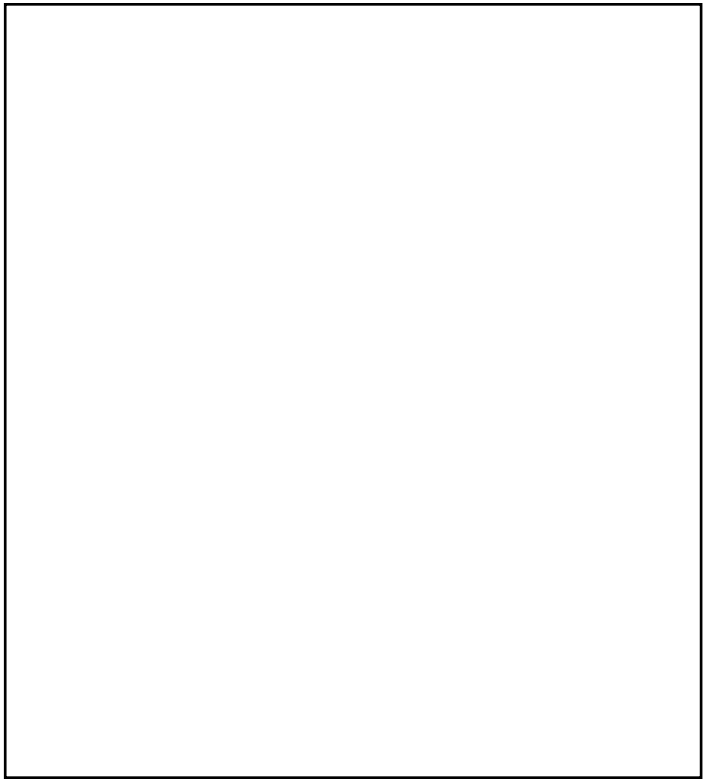
```
inspect if Damage(A) == Severe then Prep(A) else Prep(B)

value: Low
```

Advantages

- No need to know the queries before running inference
  - ✦ Allows interactively querying of the posterior distribution
- `inspect(expr)` accepts any valid BLOG expression
  - ✦ The generative model is made of BLOG expressions
  - ✦ step-by-step debugging can be implemented by recursively inspecting the generative process

Step debugging



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debugger.step()
Entering: obs Damage(First) = Severe
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Entering: Damage(First)

debugger.step()
Entering: First
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debugger.step()
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debugger.step()
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Step debugging

```
debugger.step()
Entering: obs Damage(First) = Severe

debugger.step()
Entering: Damage(First)

debugger.step()
Entering: First

debugger.step()
Entering: UniformChoice({c for City c})

debugger.runToLine(11)
Entering: case Prep(c) in
{
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                        Severe -> 0.2,
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Inspect local variables

```
debugger.inspect('c');
Inspect: c
value: B
```

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debugger.inspect('c');
Inspect: c
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debugger.inspect('Prep(c)');
Inspect: Prep(c)
value: Low
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Inspect local variables

```
debugger.inspect('c');
Inspect: c
value: B

debugger.inspect('Prep(c)');
Inspect: Prep(c)
value: Low

debugger.switchTo('Prep(B) == High')
switching to trace compatible: #175
Entering: obs Damage(First) = Severe
```

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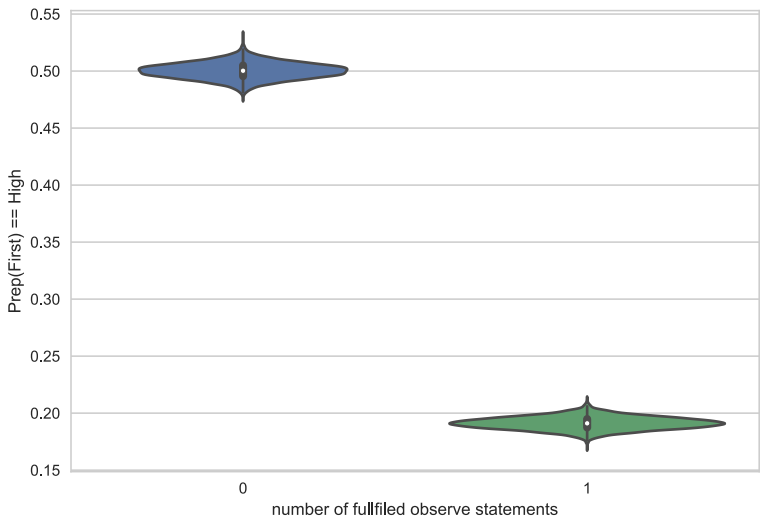
Advantages

- No need to know the queries before running inference
  - ✦ Allows interactively querying of the posterior distribution
- `inspect(expr)` accepts any valid BLOG expression
  - ✦ The generative model is made of BLOG expressions
  - ✦ step-by-step debugging can be implemented by recursively inspecting the generative process
- Evaluate impact of information for any query
  - ✦ Compute the posterior with different subsets of observations, and evaluate the expression in each sample of the posterior.

Impact of data

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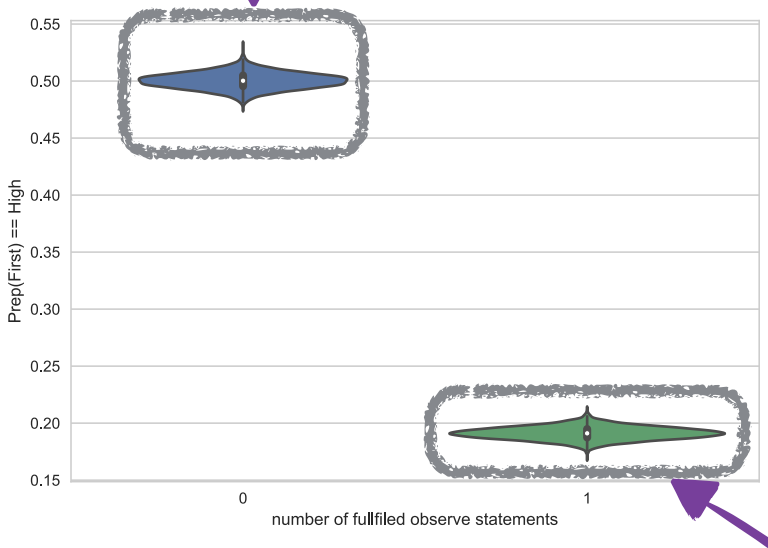
```

```
debugger.plot_query('Prep(First) == High')
```

Impact of data

Prior



Posterior

```
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```

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debugger.plot_query('Prep(First) == High')
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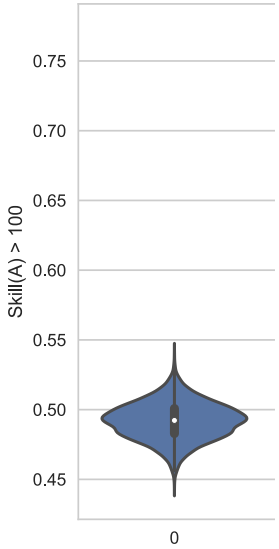
Impact of data



Bayesian Skill rating

```
debugger.plot_query('Skill(A) > 100')
```

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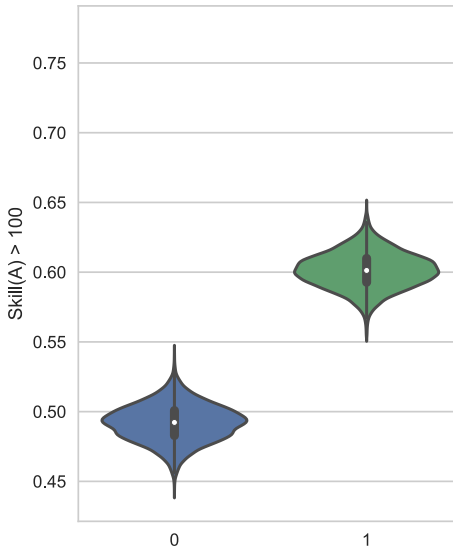


```
obs Winner(G1, A, B) = A;
```

Bayesian Skill rating

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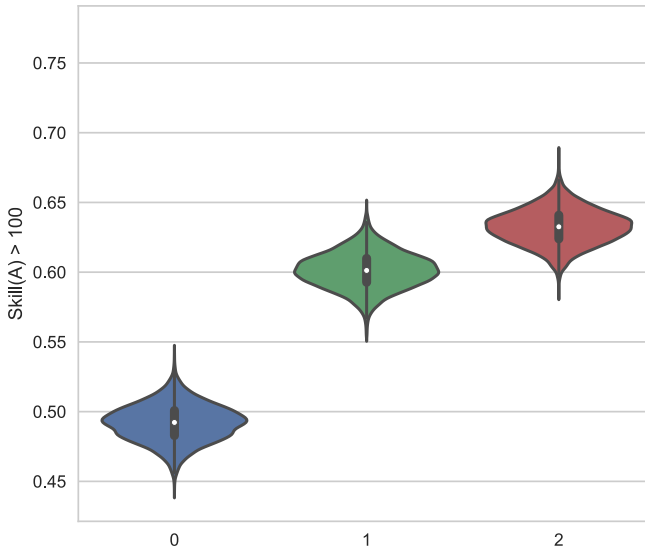


```
obs Winner(G1, A, B) = A;
obs Winner(G2, B, C) = B;
```

Bayesian Skill rating

```
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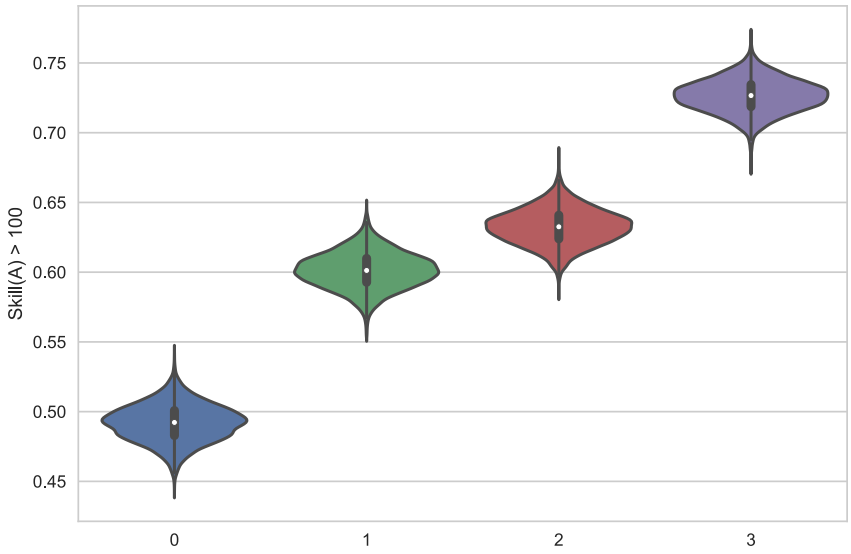


```
obs Winner(G1, A, B) = A;
obs Winner(G2, B, C) = B;
obs Winner(G3, A, C) = A;
```

Bayesian Skill rating

```
debugger.plot_query('Skill(A) > 100')
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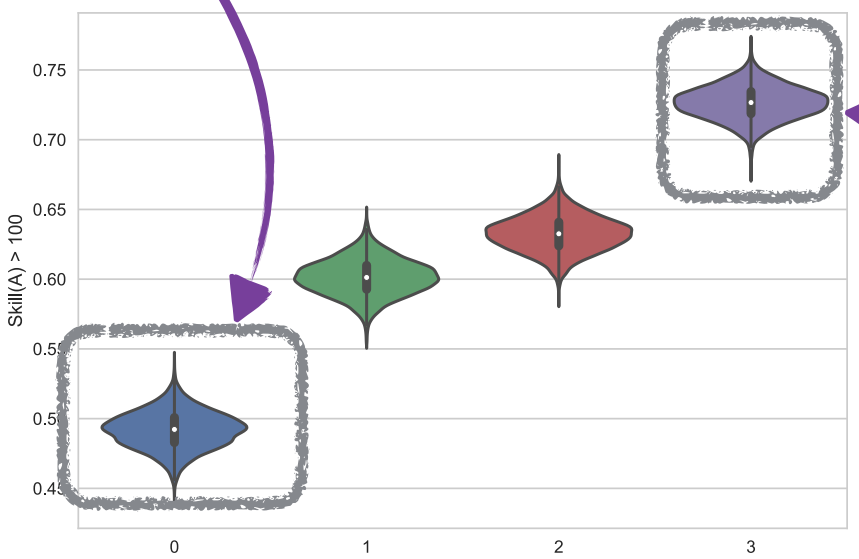
Impact of data

Prior




```
obs Winner(G1, A, B) = A;
obs Winner(G2, B, C) = B;
obs Winner(G3, A, C) = A;
```

Posterior

Bayesian Skill rating

```
debugger.plot_query('Skill(A) > 100')
```

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Final remarks

- This debugger is not meant to find bugs, but as a tool for understanding the program —the probabilistic model.
- It uses information that Bayesian PPL (e.g. anglican) uses:
  - ✦ Information regarding `sample` and `observe` are stored in ***addresses***
  - ✦ Taking subsets of observations is a fundamental step for Sequential Monte Carlo sampling
- So it can be implemented in other languages.

query Questions?

Appendix A: Bayesian Skill rating model

```
type Player;
type Game;
distinct Player A, B, C;
distinct Game G1, G2, G3;

random Real Skill(Player p) ~ Gaussian(100.0, 10.0);
random Real Performance(Player p, Game g) ~ Gaussian(Skill(p), 15.0);
random Player Winner(Game g, Player p1, Player p2) ~
                if (Performance(p1, g) > Performance(p2, g))
                   then p1
                else p2;

obs Winner(G1, A, B) = A;
obs Winner(G2, B, C) = B;
obs Winner(G3, A, C) = A;
```

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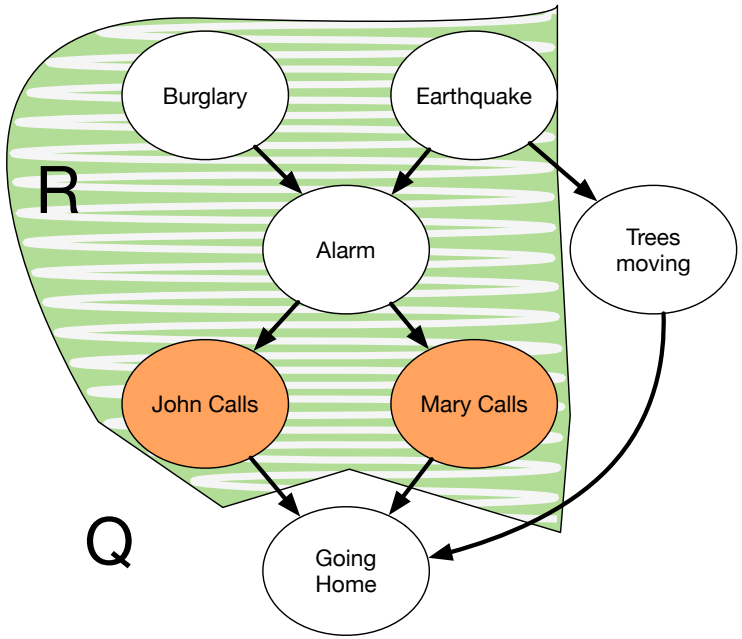
Appendix B: backward and forward inference



- Region R is learned using backward inference
- Region Q is learned lazily using forward inference