ExceLint: Automatically Finding Formula Errors in Spreadsheets

Daniel Barowy ***Emery Berger*** *Ben Zorn*

Reinhart-Rogoff

Growth in a Time of Debt

>90% debt : GDP ratio ⇒ low economic growth

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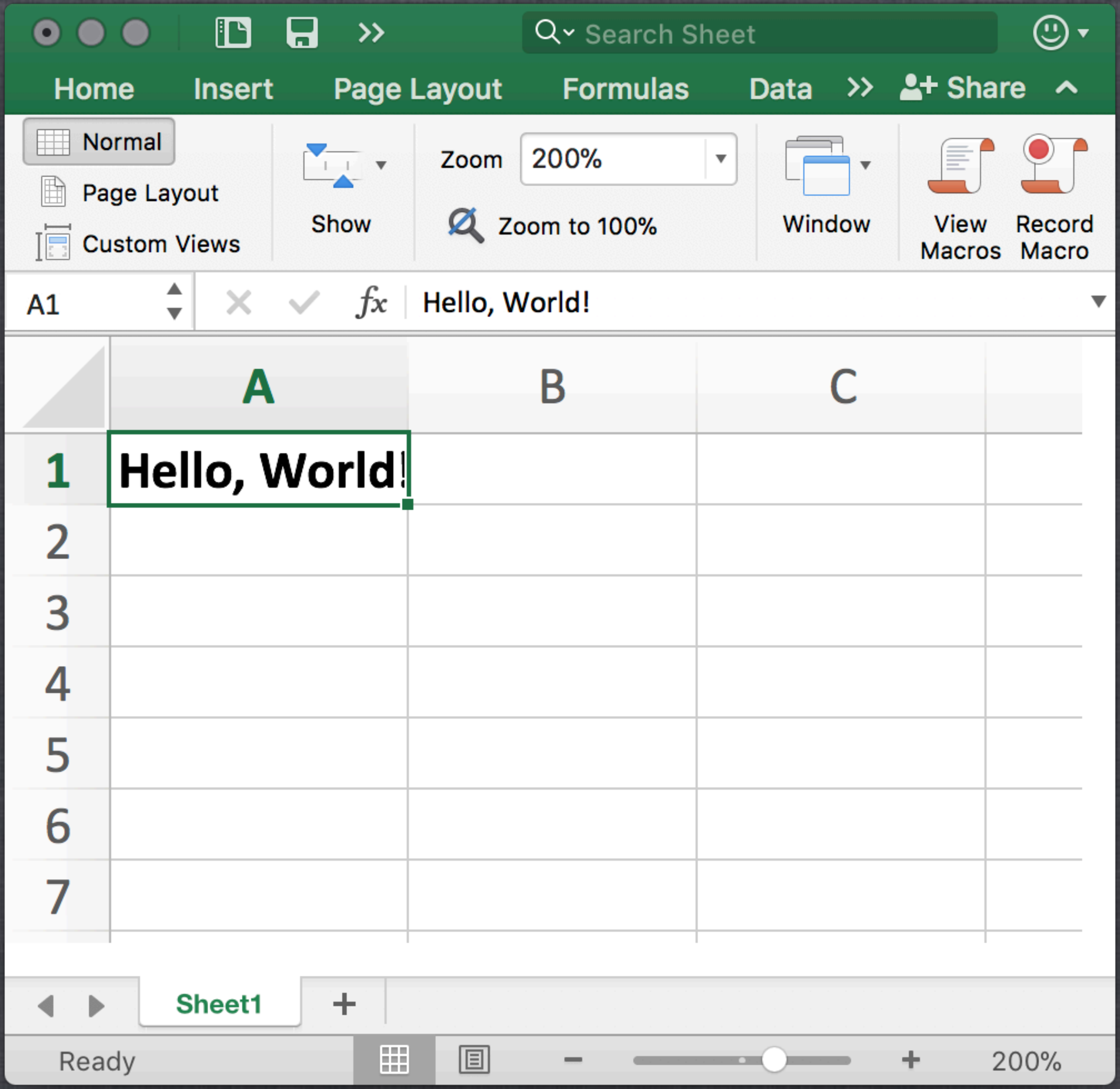
What Programming Language Did *They* Use?

Excel.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Home
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Page Layout
Formulas
Data
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Normal
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Zoom
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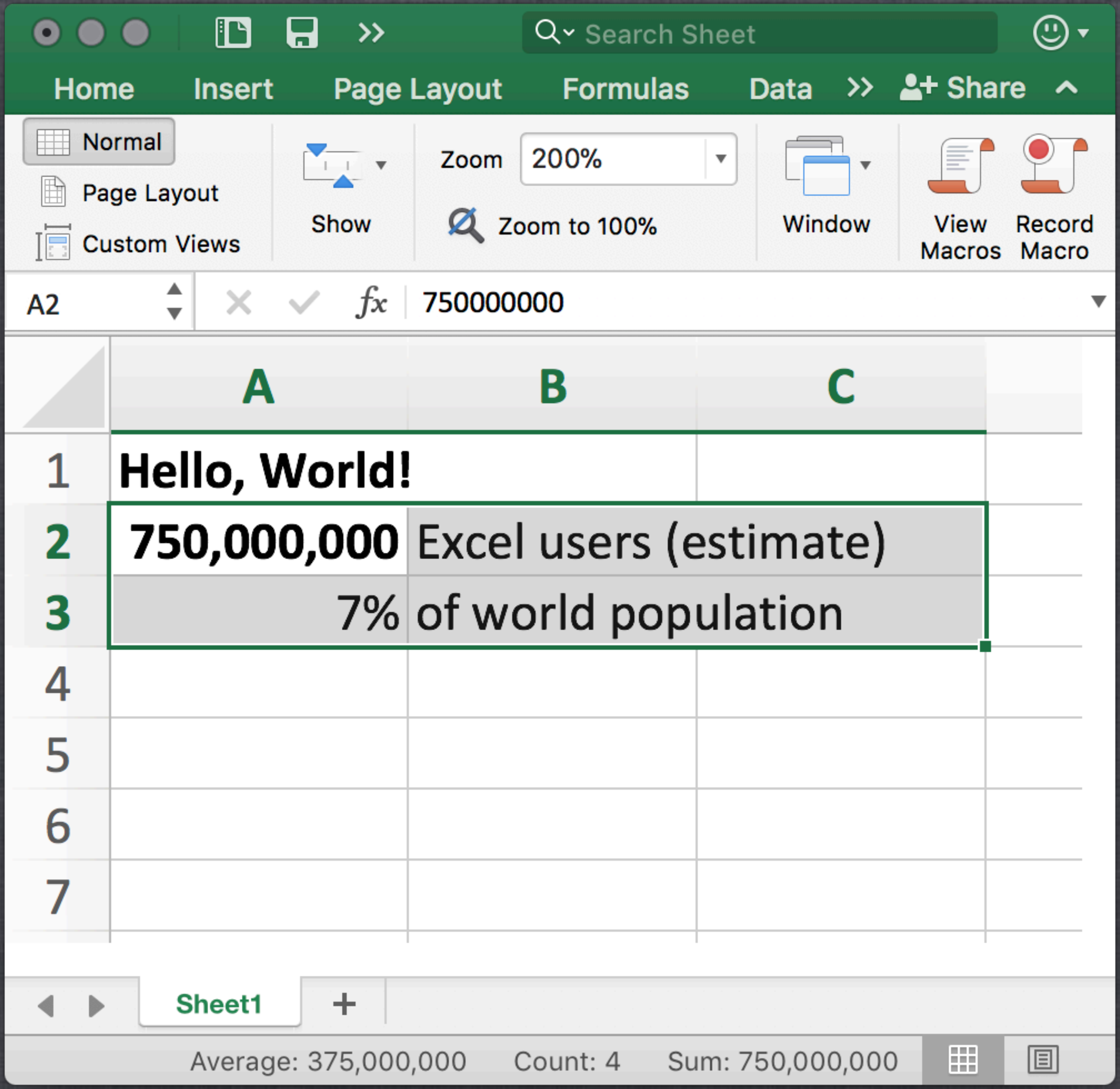
Zoom to 100%
Window
View Macros
Record Macro
A1
Hello, World!
A
B
C
1
2
3
4
5
6
7
Hello, World!
Sheet1
Ready
200%

Excel.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200%
Zoom to 100%
Window
View Macros
Record Macro
A2
750000000
A
B
C
Hello, World!
750,000,000
Excel users (estimate)
7%
of world population
Sheet1
Average: 375,000,000
Count: 4
Sum: 750,000,000

The Excel Depression

By PAUL KRUGMAN
Published: April 18, 2013 | 470 Comments

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In this age of information, math errors can lead to disaster. NASA's Mars Orbiter crashed because engineers forgot to convert to metric measurements; JPMorgan Chase's "London Whale" venture went bad in part because modelers divided by a sum instead of an average. So, did an Excel coding error destroy the economies of the Western world?

Enlarge This Image

Fred R. Conrad/The New York Times

Paul Krugman

Go to Columnist Page »

The story so far: At the beginning of 2010, two Harvard economists, Carmen Reinhart and Kenneth Rogoff, circulated a paper, "Growth in a Time of Debt," that purported to identify a critical "threshold," a tipping point, for government indebtedness. Once debt exceeds 90 percent of gross domestic product, they claimed, economic growth drops off sharply.

Ms. Reinhart and Mr. Rogoff had credibility thanks to a widely admired earlier book on the history of financial crises, and their timing was impeccable. The paper came out just after Greece went into crisis and played right into the desire of many officials to "pivot" from stimulus to austerity. As a result, the paper instantly became famous; it was, and is, surely the most influential economic analysis of recent years.

FAQ: Reinhart, Rogoff, and the Excel Error That Changed History

By Peter Coy | April 18, 2013

SEND TO kindle

Photograph by Gregor Schuster

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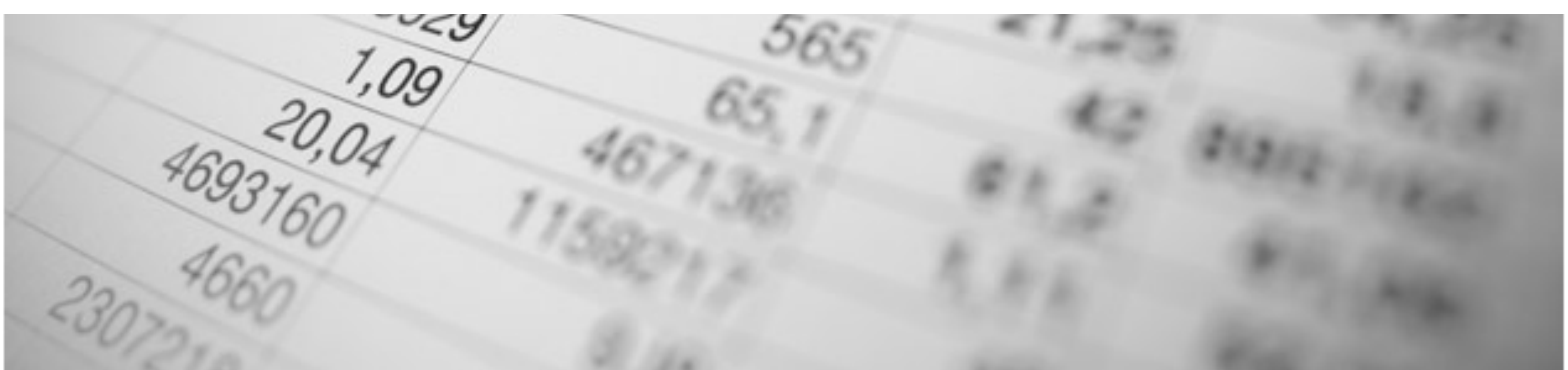
How big is this mistake?

Reinhart and Rogoff wrote in their 2010 paper that average annual growth was negative 0.1 percent in countries with episodes of gross government debt equal to 90 percent or more of GDP between 1945 and 2009. Liberal economists have been critical of their work for years (just economists being their usual cranky selves), but now three economists at UMass say Reinhart and Rogoff made several mistakes and omissions. According to the UMass scholars, the "corrected" number is positive 2.2 percent—which means GDP still grows, even when debt levels are very high.



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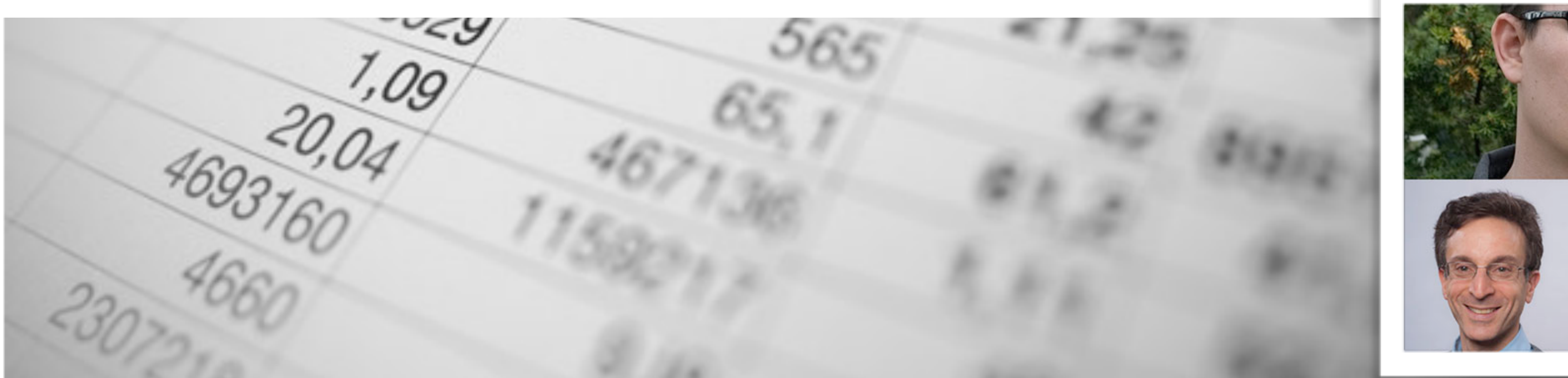

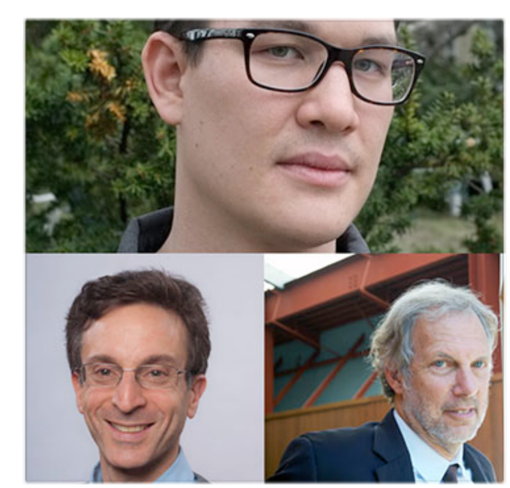

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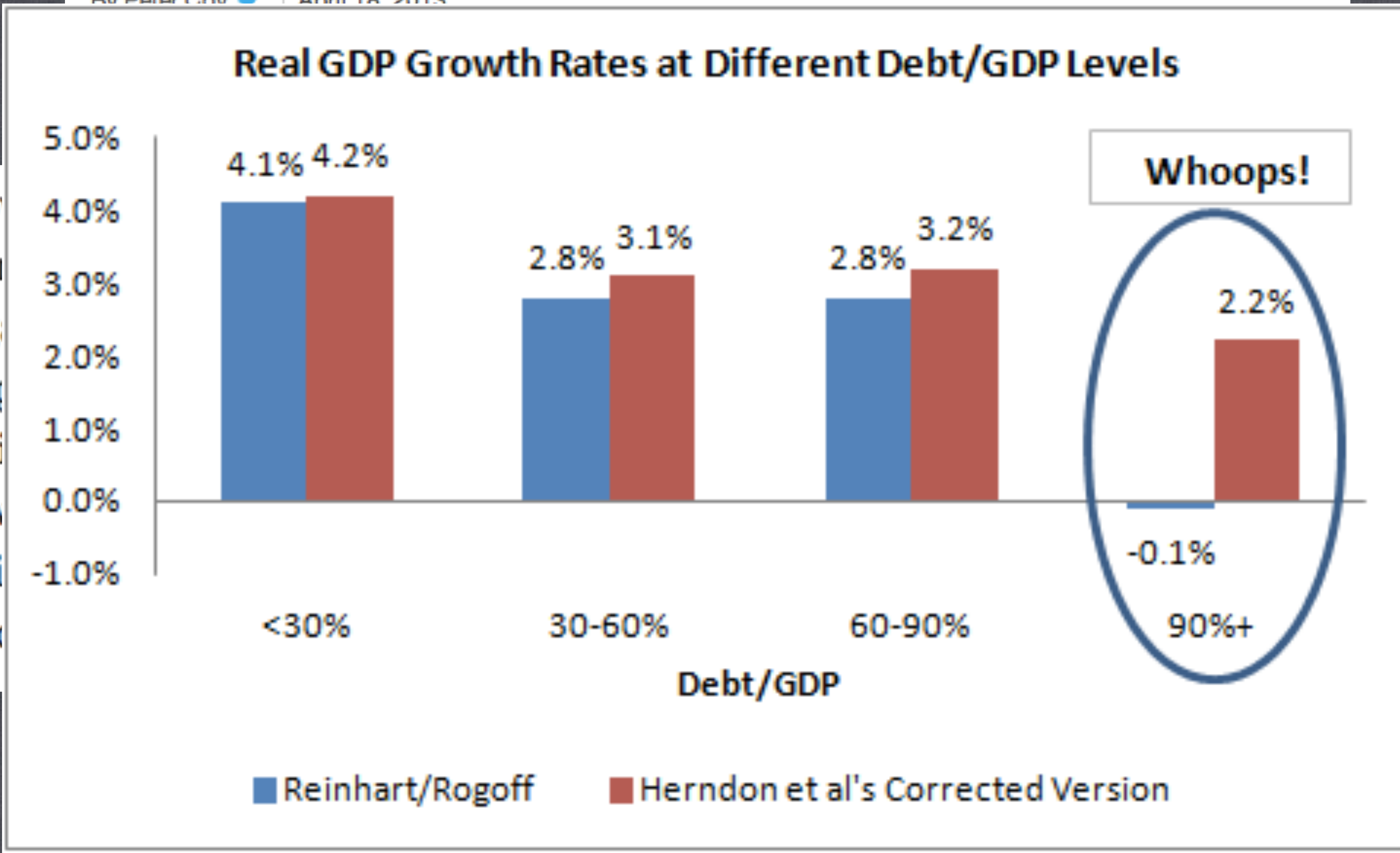

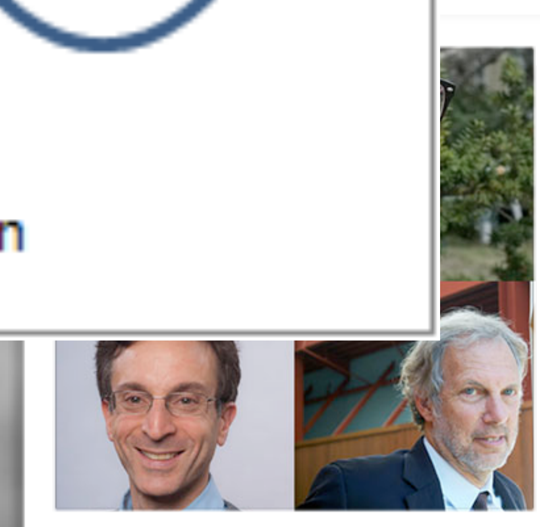

Photograph by Gregor Schuster

Sorry, Your Spreadsheet Has Errors (Almost 90% Do)

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[Forbes 2014]

State of the Art

How to Avoid Spreadsheet Errors

Like most programs, using spreadsheet software like Microsoft Excel is a manual process. That means your calculations are always susceptible to human error, such as keying in the wrong numbers or copying over the wrong amounts.

Unfortunately, every formula is unique and has a specific task, there isn't an automated tool to check for spreadsheet errors. Still, there are some steps you can take to prevent most errors from happening.

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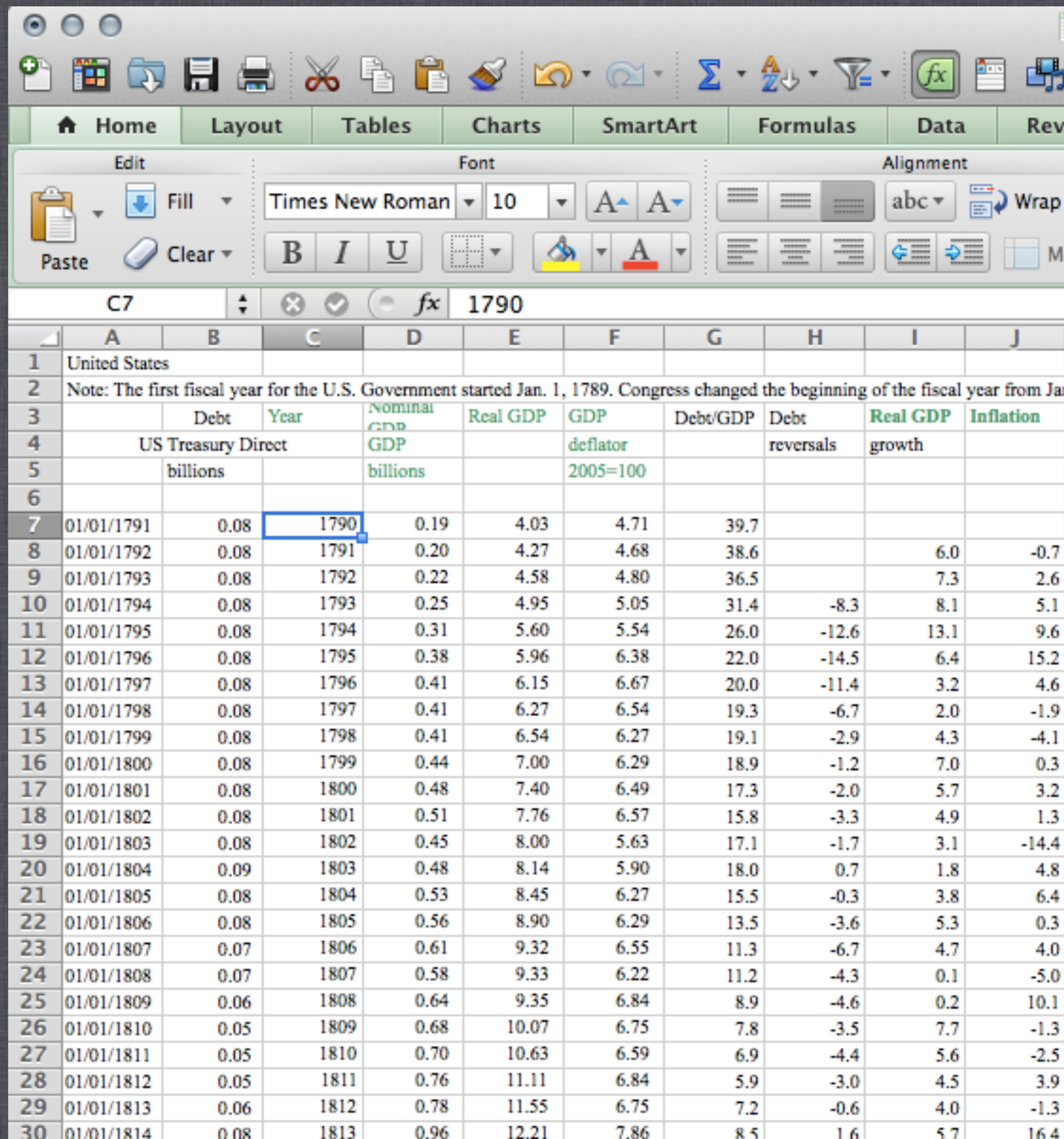
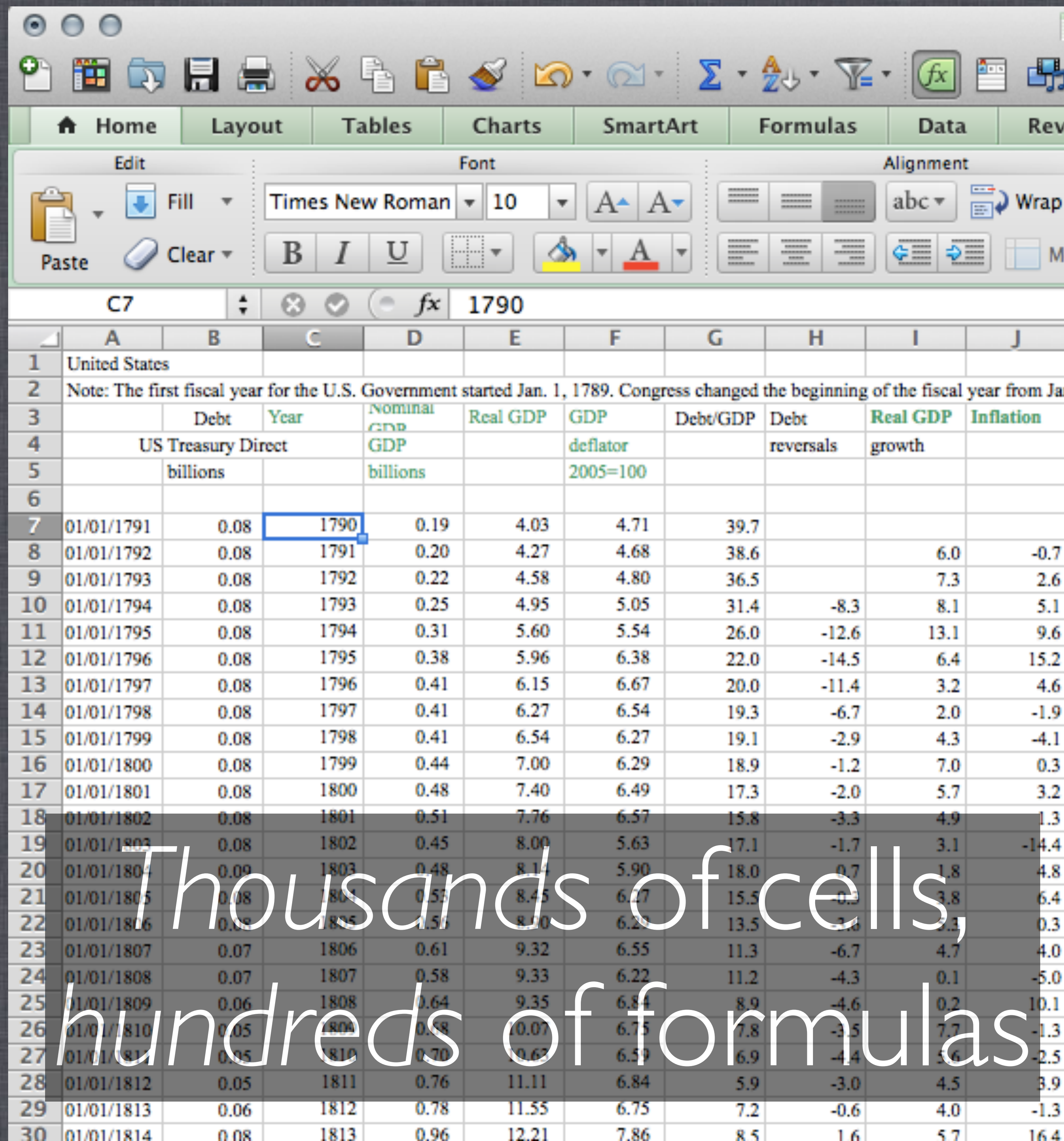
According to experts, many mistakes can be spotted early visually or through double-checking formulas.

Reinhart-Rogoff Spreadsheet



	A	B	C	D	E	F	G	H	I	J
1	United States									
2	Note: The first fiscal year for the U.S. Government started Jan. 1, 1789. Congress changed the beginning of the fiscal year from Ja									
3		Debt	Year	Nominal GDP	Real GDP	GDP	Debt/GDP	Debt	Real GDP	Inflation
4		US Treasury Direct		GDP		deflator		reversals	growth	
5		billions		billions		2005=100				
6										
7	01/01/1791	0.08	1790	0.19	4.03	4.71	39.7			
8	01/01/1792	0.08	1791	0.20	4.27	4.68	38.6		6.0	-0.7
9	01/01/1793	0.08	1792	0.22	4.58	4.80	36.5		7.3	2.6
10	01/01/1794	0.08	1793	0.25	4.95	5.05	31.4	-8.3	8.1	5.1
11	01/01/1795	0.08	1794	0.31	5.60	5.54	26.0	-12.6	13.1	9.6
12	01/01/1796	0.08	1795	0.38	5.96	6.38	22.0	-14.5	6.4	15.2
13	01/01/1797	0.08	1796	0.41	6.15	6.67	20.0	-11.4	3.2	4.6
14	01/01/1798	0.08	1797	0.41	6.27	6.54	19.3	-6.7	2.0	-1.9
15	01/01/1799	0.08	1798	0.41	6.54	6.27	19.1	-2.9	4.3	-4.1
16	01/01/1800	0.08	1799	0.44	7.00	6.29	18.9	-1.2	7.0	0.3
17	01/01/1801	0.08	1800	0.48	7.40	6.49	17.3	-2.0	5.7	3.2
18	01/01/1802	0.08	1801	0.51	7.76	6.57	15.8	-3.3	4.9	1.3
19	01/01/1803	0.08	1802	0.45	8.00	5.63	17.1	-1.7	3.1	-14.4
20	01/01/1804	0.09	1803	0.48	8.14	5.90	18.0	0.7	1.8	4.8
21	01/01/1805	0.08	1804	0.53	8.45	6.27	15.5	-0.3	3.8	6.4
22	01/01/1806	0.08	1805	0.56	8.90	6.29	13.5	-3.6	5.3	0.3
23	01/01/1807	0.07	1806	0.61	9.32	6.55	11.3	-6.7	4.7	4.0
24	01/01/1808	0.07	1807	0.58	9.33	6.22	11.2	-4.3	0.1	-5.0
25	01/01/1809	0.06	1808	0.64	9.35	6.84	8.9	-4.6	0.2	10.1
26	01/01/1810	0.05	1809	0.68	10.07	6.75	7.8	-3.5	7.7	-1.3
27	01/01/1811	0.05	1810	0.70	10.63	6.59	6.9	-4.4	5.6	-2.5
28	01/01/1812	0.05	1811	0.76	11.11	6.84	5.9	-3.0	4.5	3.9
29	01/01/1813	0.06	1812	0.78	11.55	6.75	7.2	-0.6	4.0	-1.3
30	01/01/1814	0.08	1813	0.96	12.21	7.86	8.5	1.6	5.7	16.4

Reinhart-Rogoff Spreadsheet



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20	01/01/1804	0.09	1803	0.48	8.14	5.90	18.0	[illegible]	1.8	4.8
21	01/01/1805	0.08	[illegible]	0.53	8.45	6.27	15.5	[illegible]	3.8	6.4
22	01/01/1806	[illegible]	1805	0.56	[illegible]	[illegible]	13.5	[illegible]	[illegible]	0.3
23	01/01/1807	0.07	1806	0.61	9.32	6.55	11.3	-6.7	4.7	4.0
24	01/01/1808	0.07	1807	0.58	9.33	6.22	11.2	-4.3	0.1	-5.0
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26	01/01/1810	0.05	[illegible]	[illegible]	10.07	6.75	7.8	-3.5	7.7	-1.3
27	[illegible]	0.05	1810	0.70	10.63	6.59	6.9	-4.4	5.6	-2.5
28	01/01/1812	0.05	1811	0.76	11.11	6.84	5.9	-3.0	4.5	3.9
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30	01/01/1814	0.08	1813	0.96	12.21	7.86	8.5	1.6	5.7	16.4

Thousands of cells, hundreds of formulas

ExceLint

Geospatial static analysis

to reveal deep structure of sheets

ExceLint

Geospatial static analysis
to reveal deep structure of sheets

Leverages 2-D character
of spreadsheets

ExceLint

Geospatial static analysis
to reveal deep structure of sheets

Leverages 2-D character
of spreadsheets

Identifies likely errors + fixes

ExceLint

	D	E	F	G
5	Week3	Week 4	Total Hours	Overtime Hrs
6	5.25	8.58	34.33	0.00
7	20.50	17.83	53.50	0.00
8	16.00	16.83	50.50	0.00
9	43.00	41.17	123.50	5.50
10	48.00	44.00	132.00	12.00
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5	*Week3*	*Week 4*	*Total Hours*	*Overtime Hrs*
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Colors quickly identify likely errors

ExceLint

	E	F	G
5	Week 4	Total Hours	Overtime Hrs
6	8.58	34.33	0.00
7	17.83	53.50	0.00
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5	Week 4	Total Hours	Overtime Hrs
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Matches bugs with proposed “fixes”

Geospatial static analysis

Instead of ASTs, etc. — Vector-based IR

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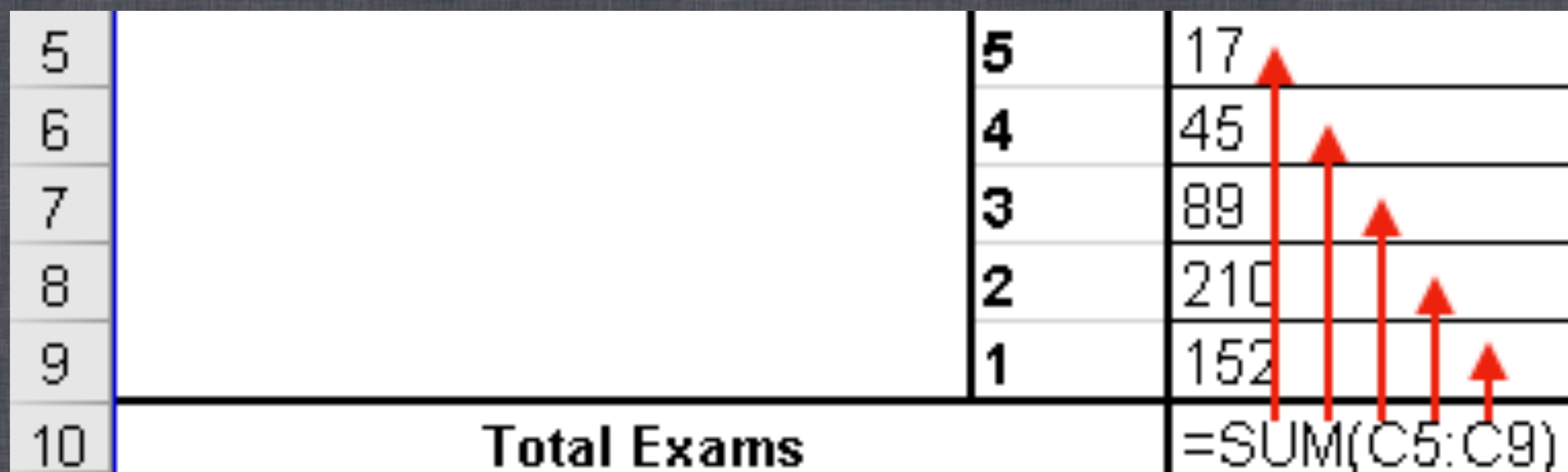

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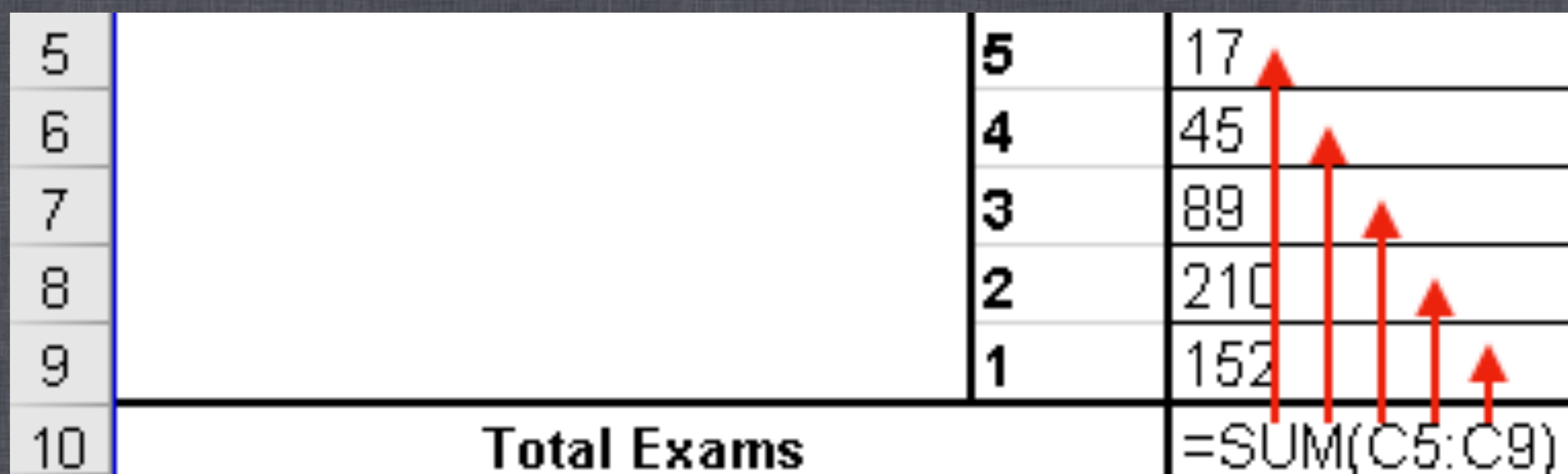

from → to $\langle\, x,\ y,\ z, dx, dy, dz \,\rangle$

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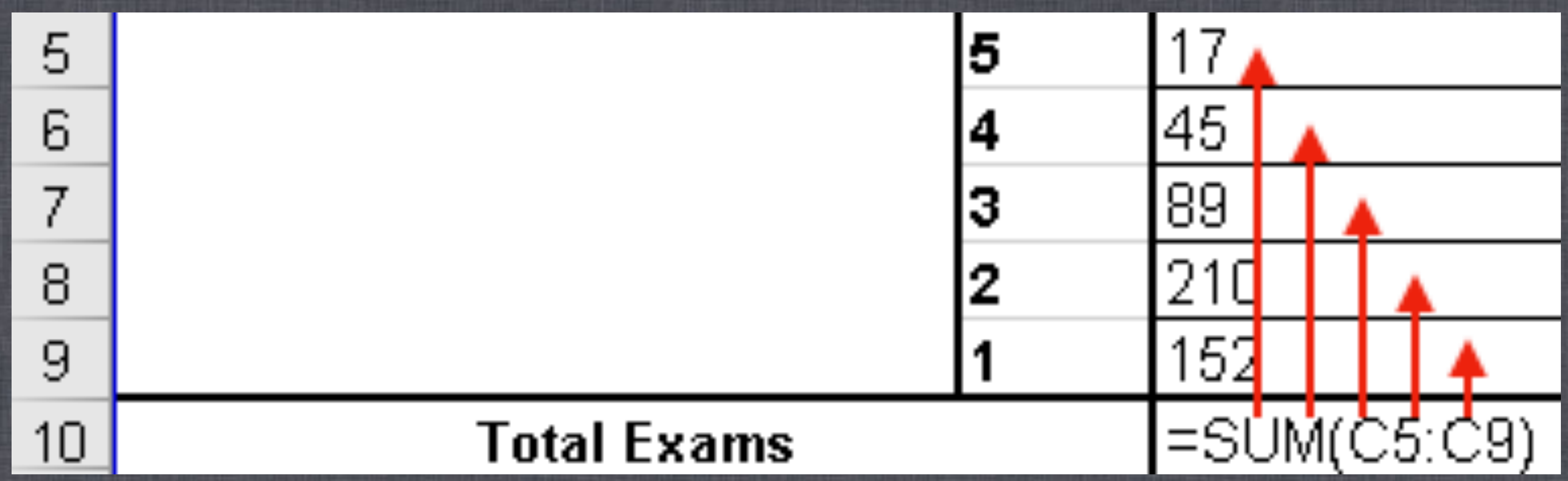

from → to ⟨ x, y, z, dx, dy, dz ⟩

1. converts each cell to reference vectors: unifying dependence & spatial relationships

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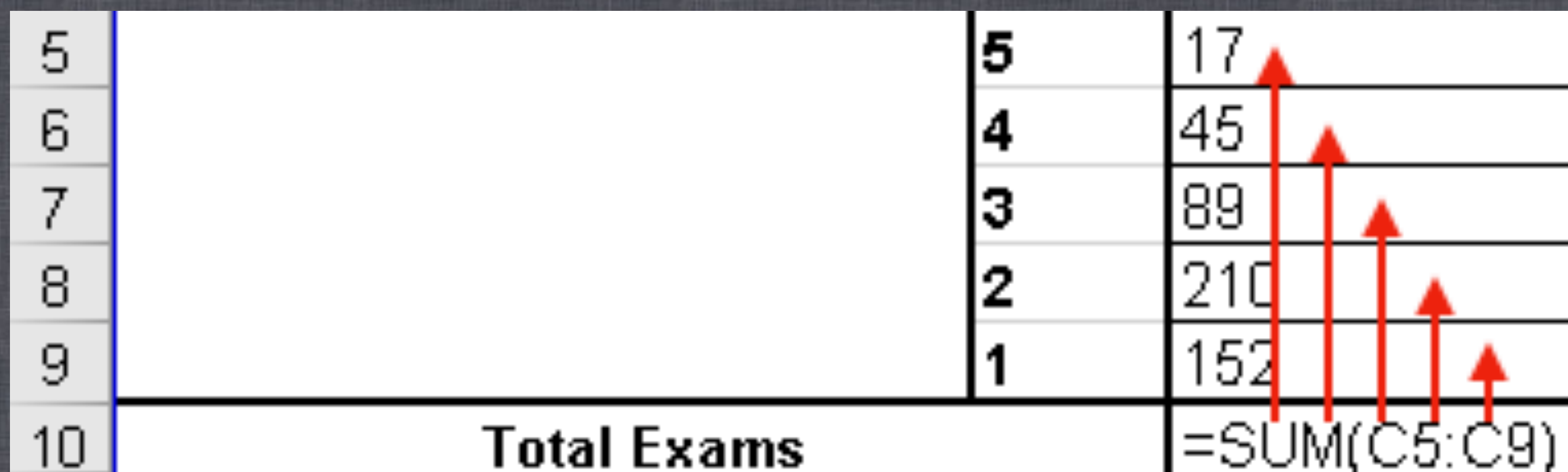

from → to $\langle$ x, y, z, dx, dy, dz $\rangle$

1. converts each cell to reference vectors: unifying dependence & spatial relationships
2. combine vectors to form “fingerprints”

Spreadsheet fingerprints

	A	B	C	D	E	F	G
6	f_1	f_2	f_2	f_2	f_2	f_3	f_5
7	f_1	f_2	f_2	f_2	f_2	f_4	f_5
8	f_1	f_2	f_2	f_2	f_2	f_4	f_5
9	f_1	f_2	f_2	f_2	f_2	f_4	f_5
10	f_1	f_2	f_2	f_2	f_2	f_4	f_5
11	f_1	f_2	f_2	f_2	f_2	f_4	f_6

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11	f_1	f_2	f_2	f_2	f_2	f_4	f_6

Data, strings and whitespace also have fingerprints

Spreadsheet fingerprints

	A	B	C	D	E	F	G
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9	f_1	f_2	f_2	f_2	f_2	f_4	f_5
10	f_1	f_2	f_2	f_2	f_2	f_4	f_5
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Data, strings and whitespace also have fingerprints

Easy for a human to see *likely errors*

Spreadsheet fingerprints

	A	B	C	D	E	F	G
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9	f_1	f_2	f_2	f_2	f_2	f_4	f_5
10	f_1	f_2	f_2	f_2	f_2	f_4	f_5
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What about *fixes?*

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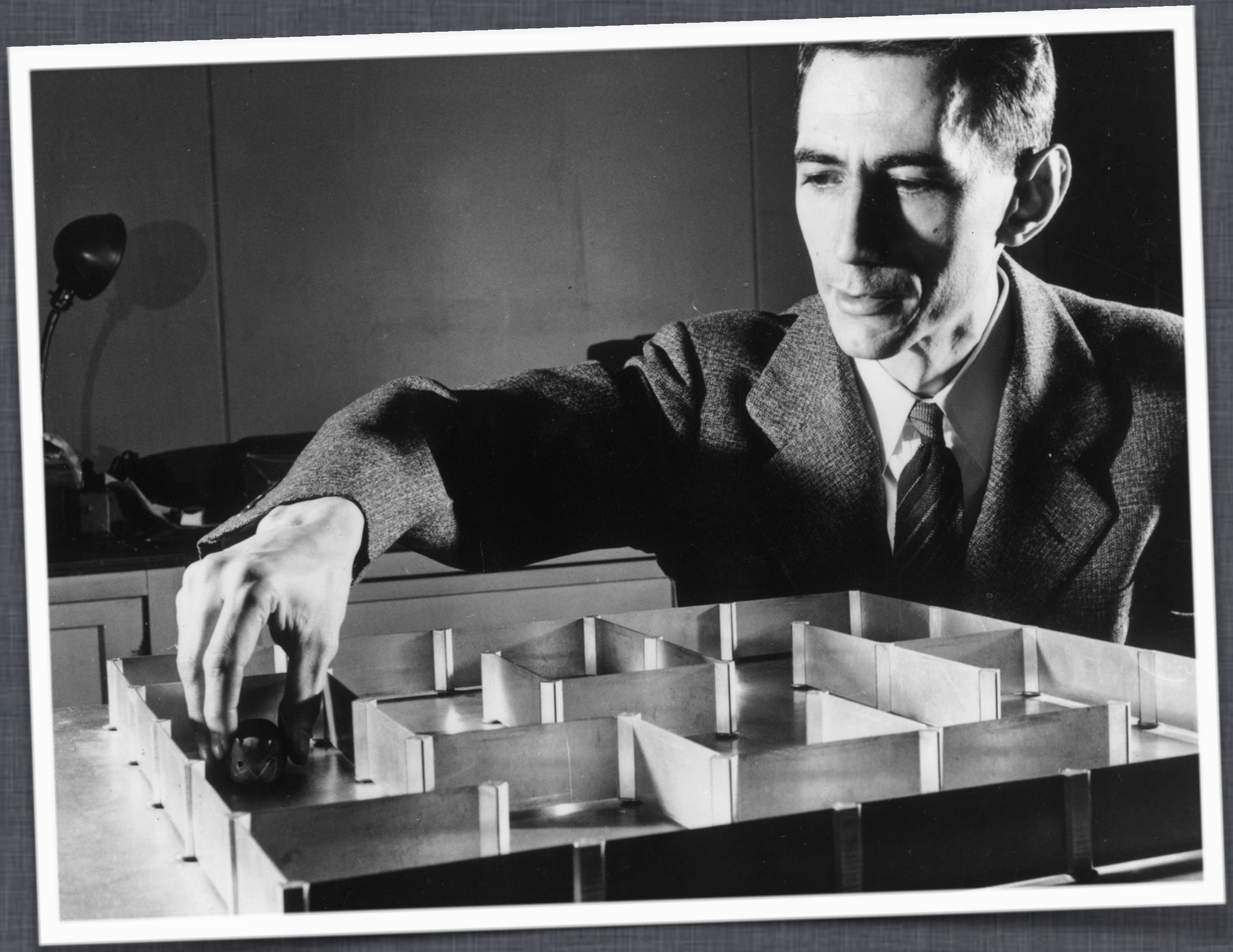
7	20.50	17.83	53.50	0.00
8	16.00	16.83	50.50	0.00
9	43.00	41.17	123.50	5.50
10	48.00	44.00	132.00	12.00
11	38.50	35.50	106.50	5.00

	E	F	G
5	Week 4	Total Hours	Overtime Hrs
6	8.58	34.33	0.00
7	17.83	53.50	0.00
8	16.83	50.50	0.00
9	41.17	123.50	5.50
10	44.00	132.00	12.00
11	35.50	106.50	5.00

Rectangular Decomposition

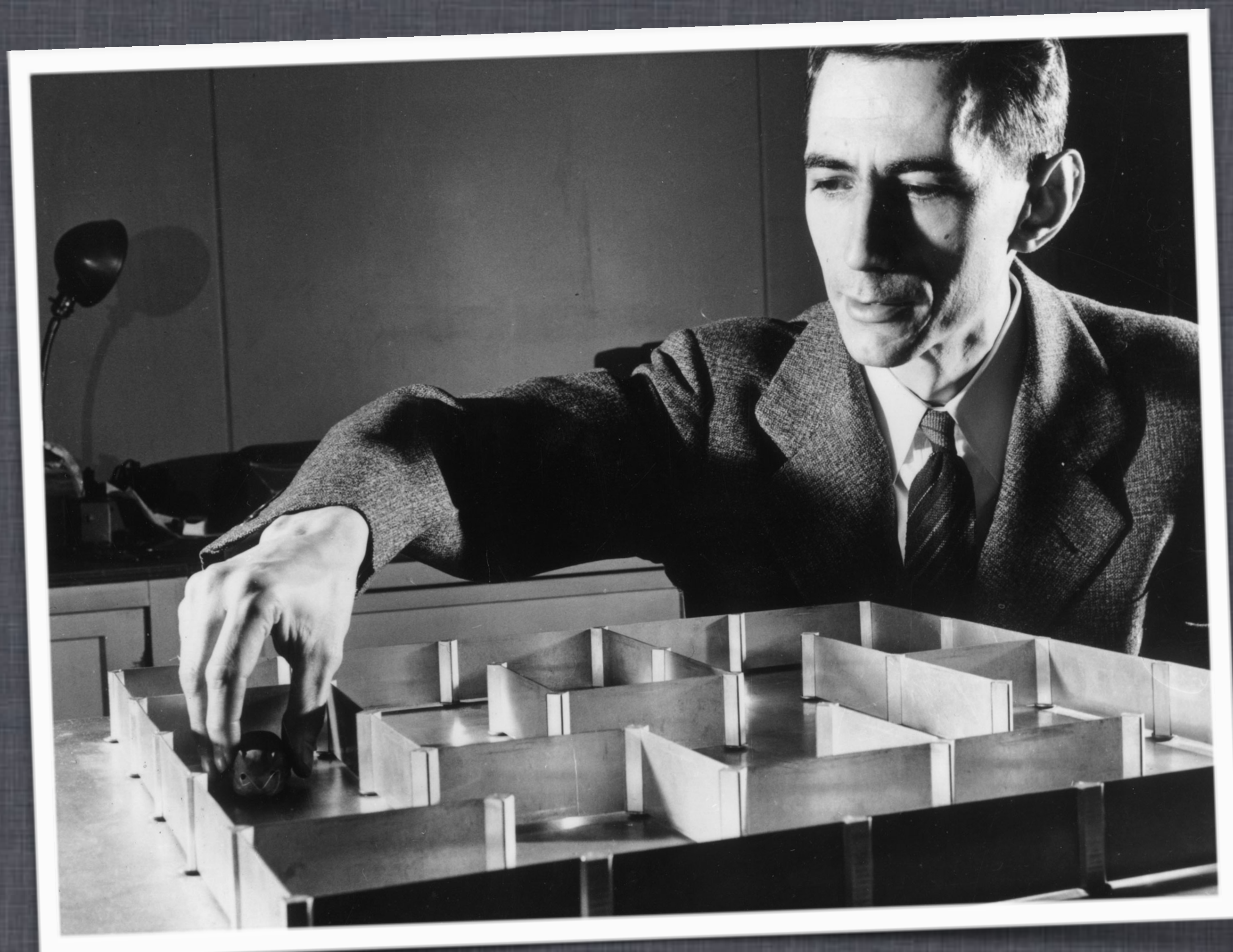
Rectangular Decomposition



$$H = -\sum_i p_i \log_b p_i$$

Entropy as proxy for geometric complexity

Rectangular Decomposition

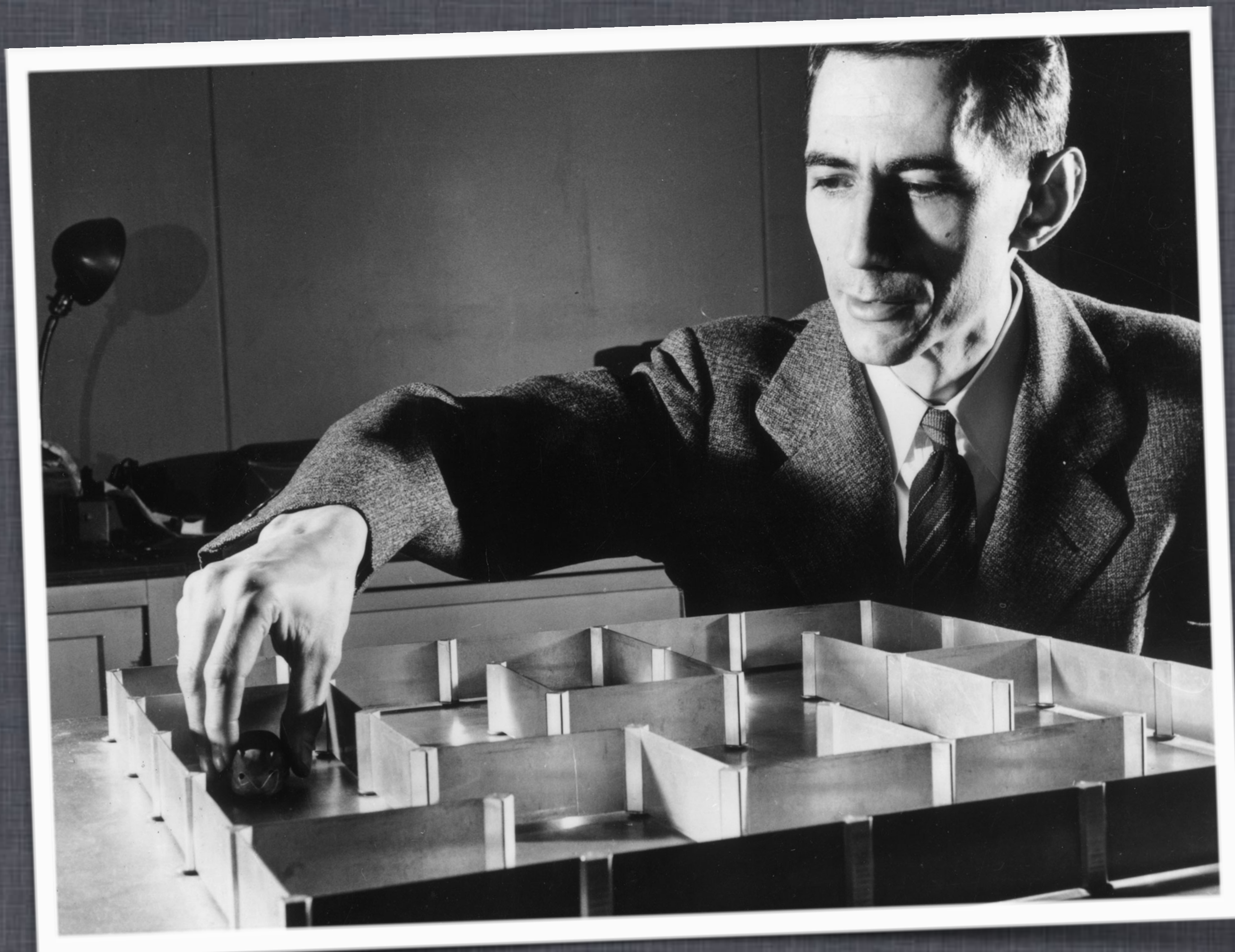


$$H = -\sum_i p_i \log_b p_i$$

Entropy as proxy for geometric complexity

Good decomposition = low entropy

Rectangular Decomposition



$$H = -\sum_i p_i \log_b p_i$$

Entropy as proxy for geometric complexity

Good decomposition = low entropy

Bad decomposition = high entropy

1st part: Binary minimal entropy decomposition

	A	B	C	D	E	F	G
6	f_1	f_2	f_2	f_2	f_2	f_3	f_5
7	f_1	f_2	f_2	f_2	f_2	f_4	f_5
8	f_1	f_2	f_2	f_2	f_2	f_4	f_5
9	f_1	f_2	f_2	f_2	f_2	f_4	f_5
10	f_1	f_2	f_2	f_2	f_2	f_4	f_5
11	f_1	f_2	f_2	f_2	f_2	f_4	f_6

Algorithm: Recursively subdivide spreadsheet by split that minimizes sum total Shannon entropy, until splits contain only equal value fingerprints.

1st part: Binary minimal entropy decomposition

	A	B	C	D	E	F	G
6	f_1	f_2	f_2	f_2	f_2	f_3	f_5
7	f_1	f_2	f_2	f_2	f_2	f_4	f_5
8	f_1	f_2	f_2	f_2	f_2	f_4	f_5
9	f_1	f_2	f_2	f_2	f_2	f_4	f_5
10	f_1	f_2	f_2	f_2	f_2	f_4	f_5
11	f_1	f_2	f_2	f_2	f_2	f_4	f_6

H(left) + H(right)

Algorithm: Recursively subdivide spreadsheet by split that minimizes sum total Shannon entropy, until splits contain only equal value fingerprints.

1st part: Binary minimal entropy decomposition

	A	B	C	D	E	F	G
6	f_1	f_2	f_2	f_2	f_2	f_3	f_5
7	f_1	f_2	f_2	f_2	f_2	f_4	f_5
8	f_1	f_2	f_2	f_2	f_2	f_4	f_5
9	f_1	f_2	f_2	f_2	f_2	f_4	f_5
10	f_1	f_2	f_2	f_2	f_2	f_4	f_5
11	f_1	f_2	f_2	f_2	f_2	f_4	f_6

H(left) + H(right)

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1st part: Binary minimal entropy decomposition

	A	B	C	D	E	F	G
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7	f_1	f_2	f_2	f_2	f_2	f_4	f_5
8	f_1	f_2	f_2	f_2	f_2	f_4	f_5
9	f_1	f_2	f_2	f_2	f_2	f_4	f_5
10	f_1	f_2	f_2	f_2	f_2	f_4	f_5
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9	f_1	f_2	f_2	f_2	f_2	f_4	f_5
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9	f_1	f_2	f_2	f_2	f_2	f_4	f_5
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	A	B	C	D	E	F	G
6	f_1	f_2	f_2	f_2	f_2	f_3	f_5
7	f_1	f_2	f_2	f_2	f_2	f_4	f_5
8	f_1	f_2	f_2	f_2	f_2	f_4	f_5
9	f_1	f_2	f_2	f_2	f_2	f_4	f_5
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	A	B	C	D	E	F	G
6	f_1	f_2	f_2	f_2	f_2	f_3	f_5
7	f_1	f_2	f_2	f_2	f_2	f_4	f_5
8	f_1	f_2	f_2	f_2	f_2	f_4	f_5
9	f_1	f_2	f_2	f_2	f_2	f_4	f_5
10	f_1	f_2	f_2	f_2	f_2	f_4	f_5
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	A	B	C	D	E	F	G
6	f_1	f_2	f_2	f_2	f_2	f_3	f_5
7	f_1	f_2	f_2	f_2	f_2	f_4	f_5
8	f_1	f_2	f_2	f_2	f_2	f_4	f_5
9	f_1	f_2	f_2	f_2	f_2	f_4	f_5
10	f_1	f_2	f_2	f_2	f_2	f_4	f_5
11	f_1	f_2	f_2	f_2	f_2	f_4	f_6

H(top) + H(bottom)

Algorithm: Recursively subdivide spreadsheet by split that minimizes sum total Shannon entropy, until splits contain only equal value fingerprints.

1st part: Binary minimal entropy decomposition

	A	B	C	D	E	F	G
6	f_1	f_2	f_2	f_2	f_2	f_3	f_5
7	f_1	f_2	f_2	f_2	f_2	f_4	f_5
8	f_1	f_2	f_2	f_2	f_2	f_4	f_5
9	f_1	f_2	f_2	f_2	f_2	f_4	f_5
10	f_1	f_2	f_2	f_2	f_2	f_4	f_5
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	A	B	C	D	E	F	G
6	f_1	f_2	f_2	f_2	f_2	f_3	f_5
7	f_1	f_2	f_2	f_2	f_2	f_4	f_5
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7	f_1	f_2	f_2	f_2	f_2	f_4	f_5
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	A	B	C	D	E	F	G
6	f_1	f_2	f_2	f_2	f_2	f_3	f_5
7	f_1	f_2	f_2	f_2	f_2	f_4	f_5
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7	f_1	f_2	f_2	f_2	f_2	f_4	f_5
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6	f_1	f_2	f_2	f_2	f_2	f_3	f_5
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Algorithm: Recursively subdivide spreadsheet by split that minimizes sum total Shannon entropy, until splits contain only equal value fingerprints.

2nd part: Minimal entropy ranking

	A	B	C	D	E	F	G
6	f_1	f_2	f_2	f_2	f_2	f_3	f_5
7	f_1	f_2	f_2	f_2	f_2	f_4	f_5
8	f_1	f_2	f_2	f_2	f_2	f_4	f_5
9	f_1	f_2	f_2	f_2	f_2	f_4	f_5
10	f_1	f_2	f_2	f_2	f_2	f_4	f_5
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Every pair of rectangles that, when merged, remains rectangular, is a "potential fix."

2nd part: Minimal entropy ranking

	A	B	C	D	E	F	G
6	f_1	f_2	f_2	f_2	f_2	f_3	f_5
7	f_1	f_2	f_2	f_2	f_2	f_4	f_5
8	f_1	f_2	f_2	f_2	f_2	f_4	f_5
9	f_1	f_2	f_2	f_2	f_2	f_4	f_5
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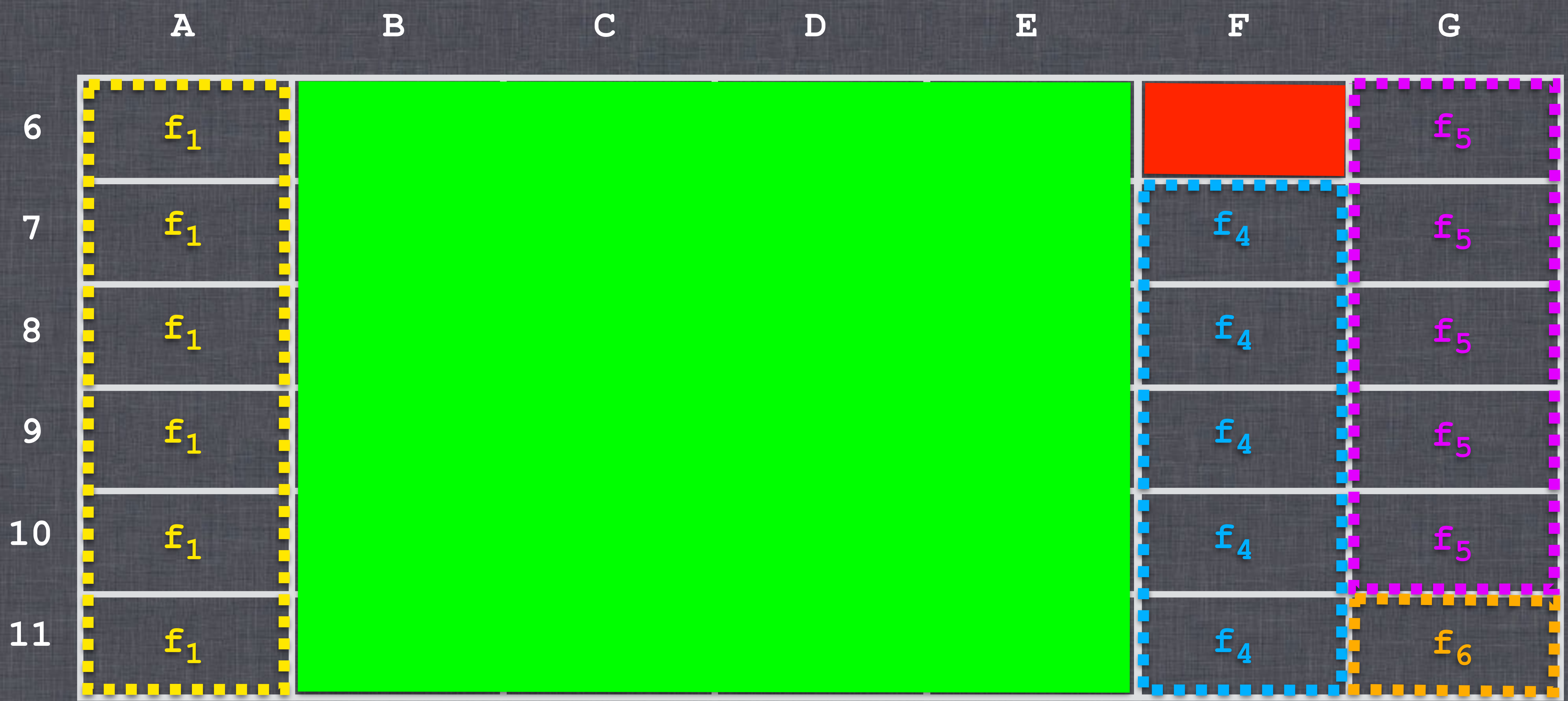

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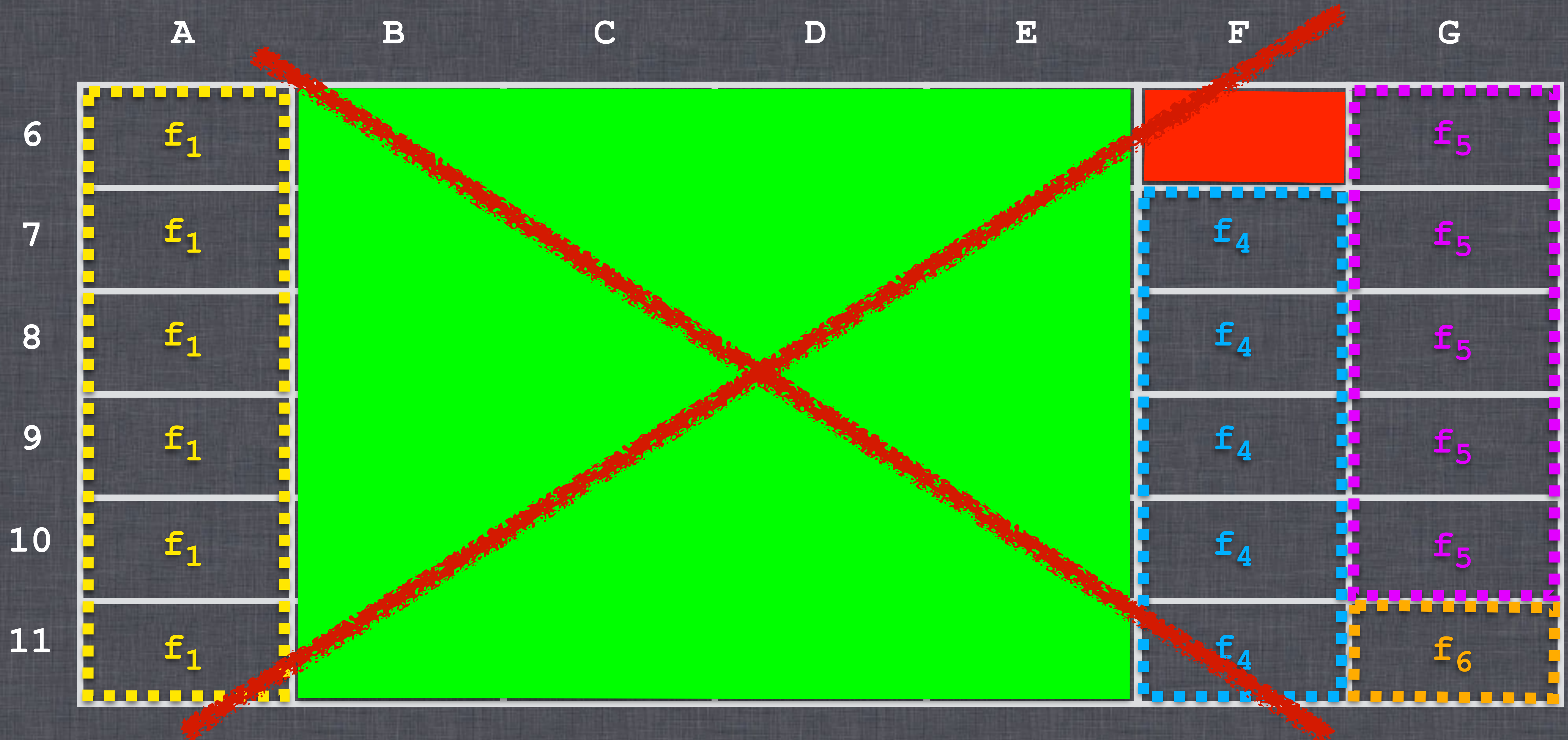
2nd part: Minimal entropy ranking



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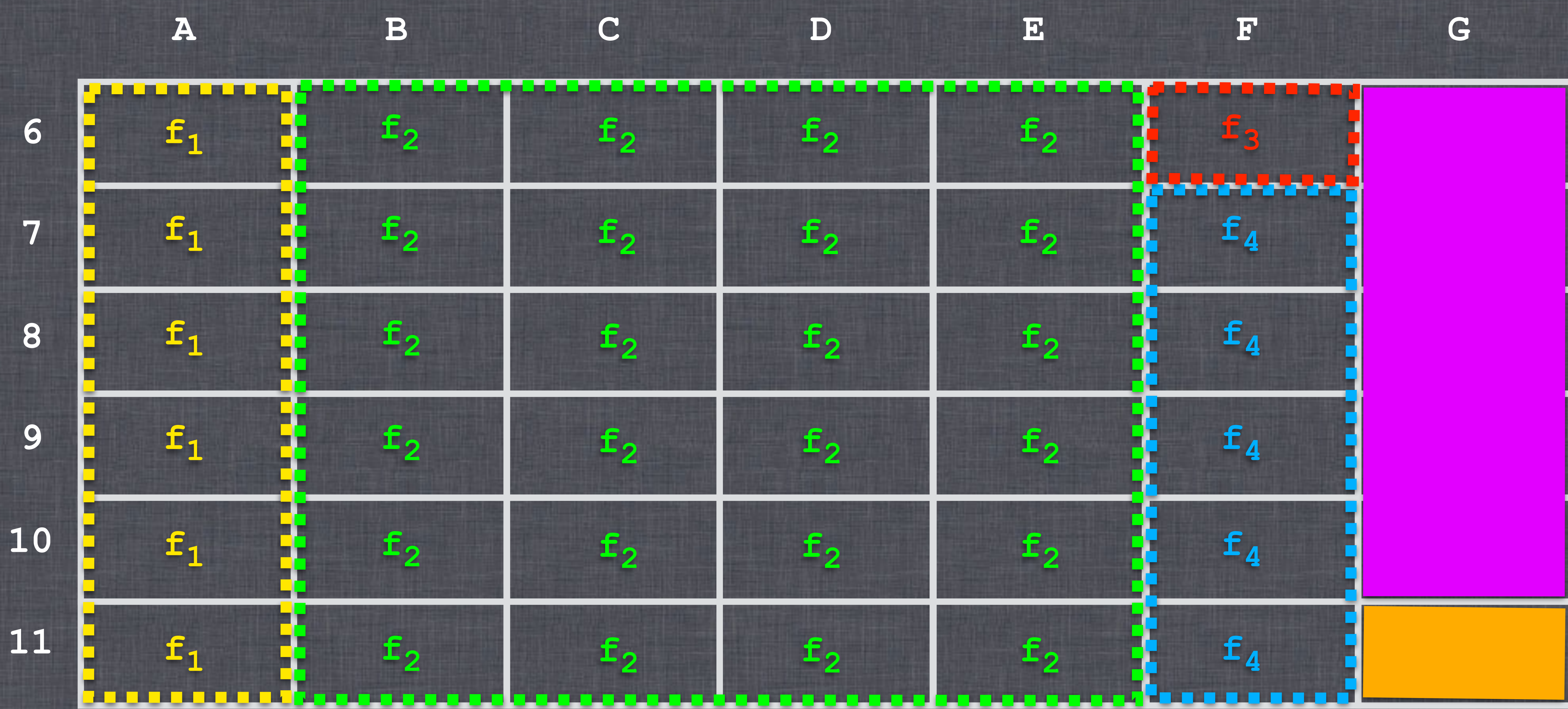
2nd part: Minimal entropy ranking



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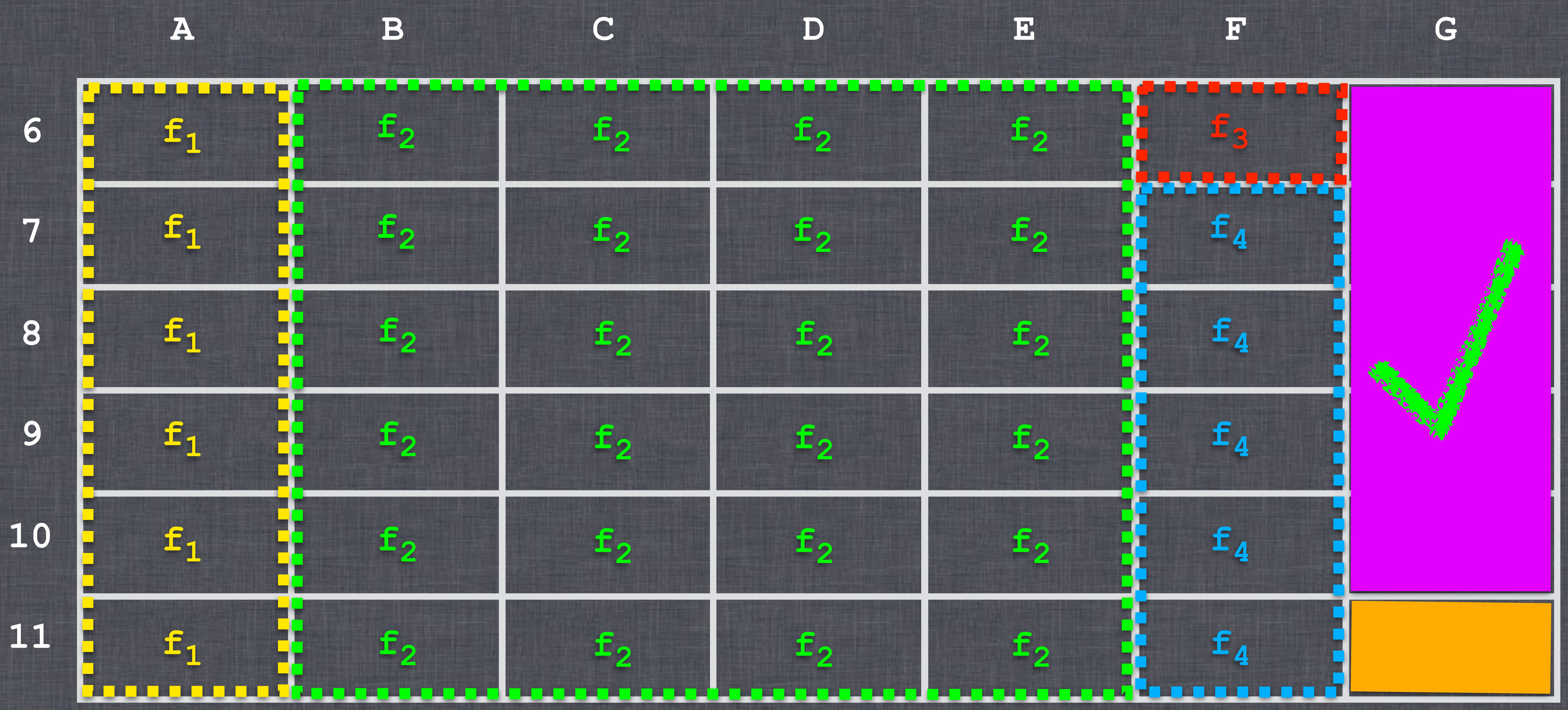
2nd part: Minimal entropy ranking



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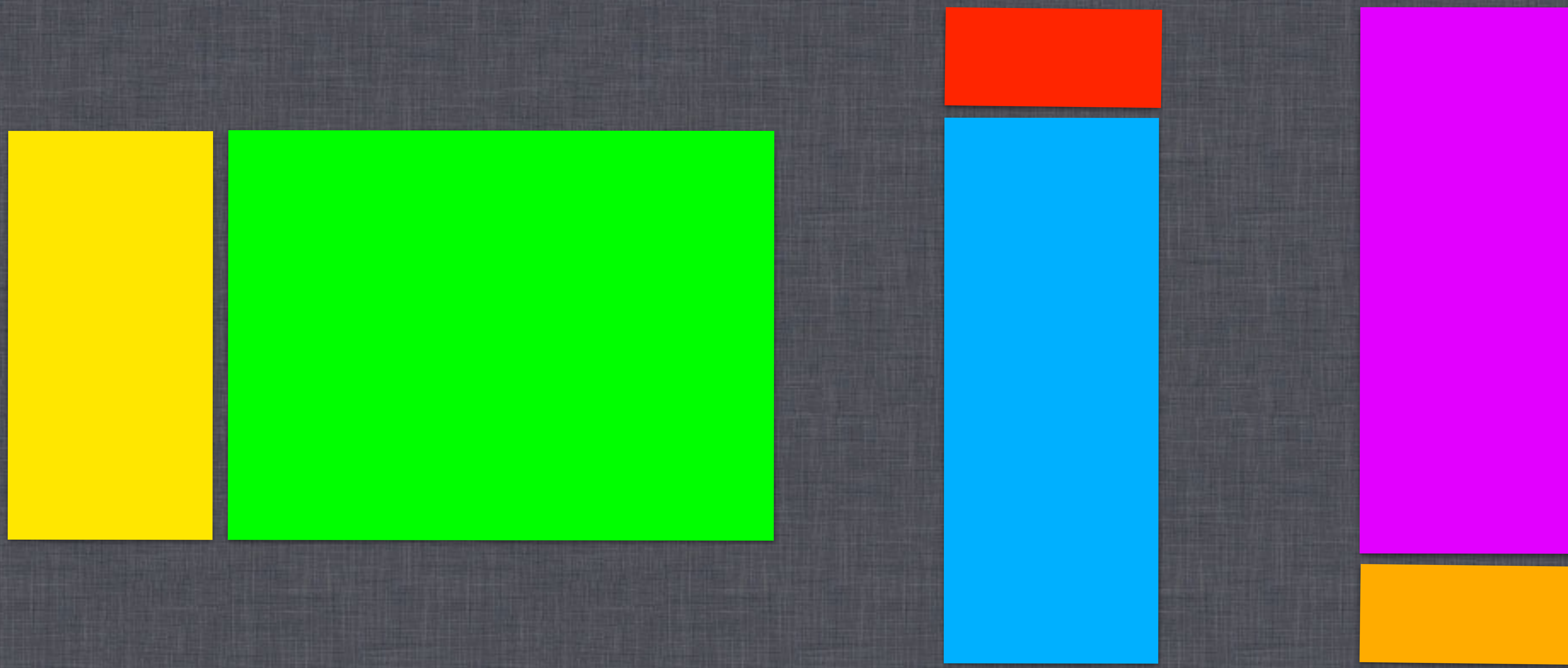
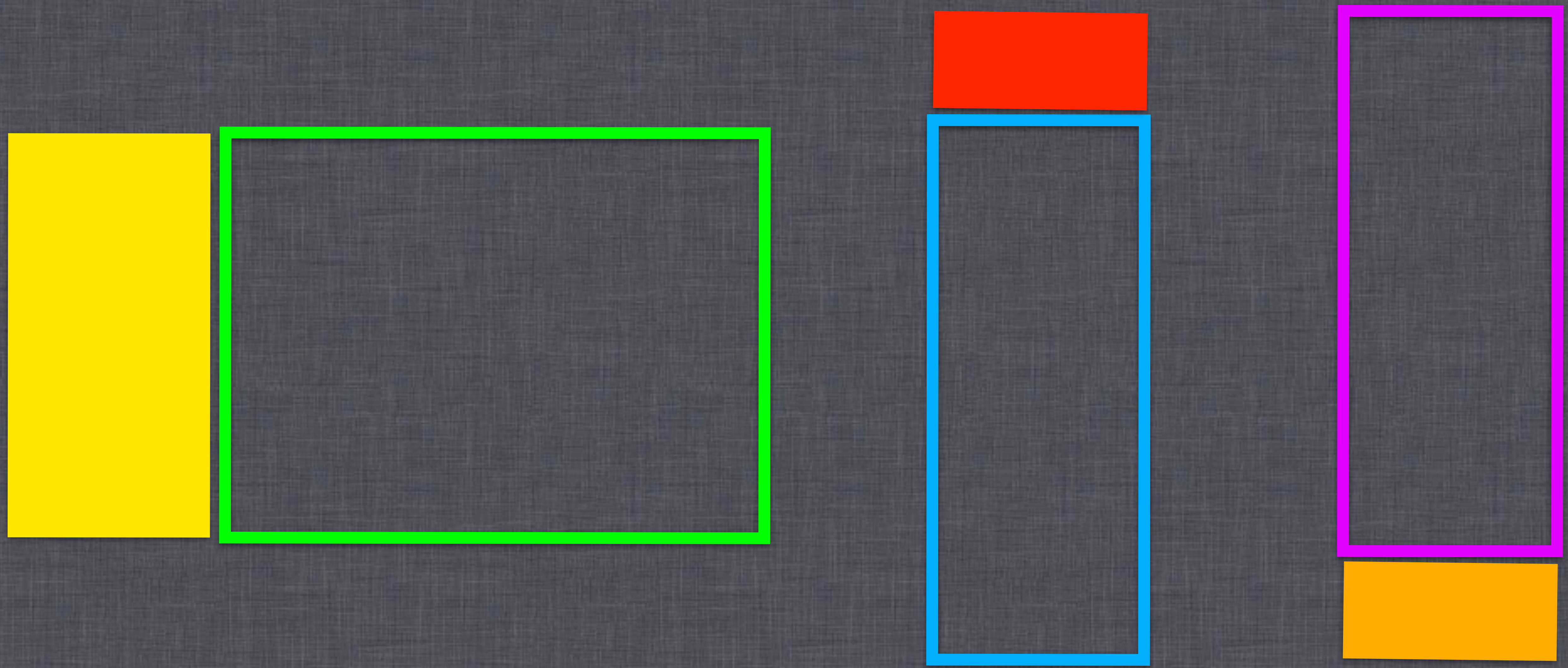
2nd part: Minimal entropy ranking

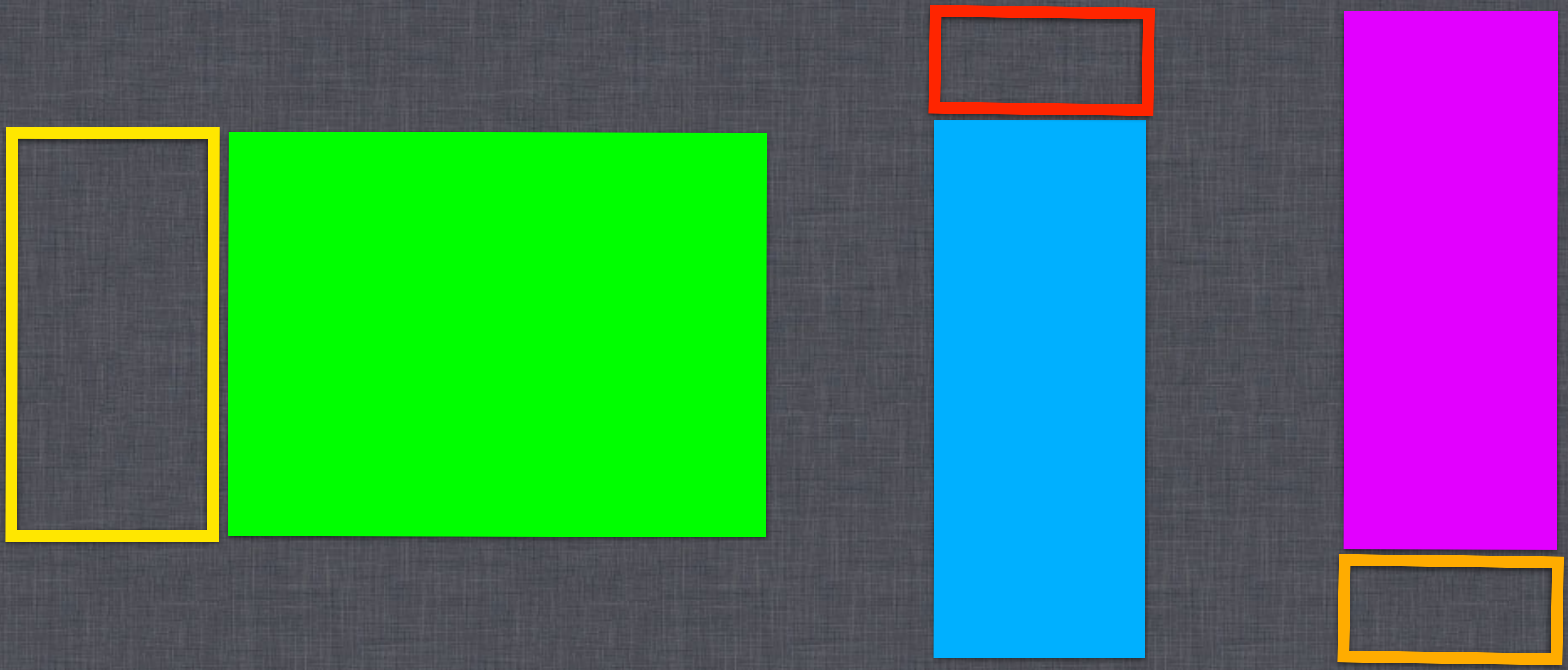


2nd part: Minimal entropy ranking



Small part is the "error"

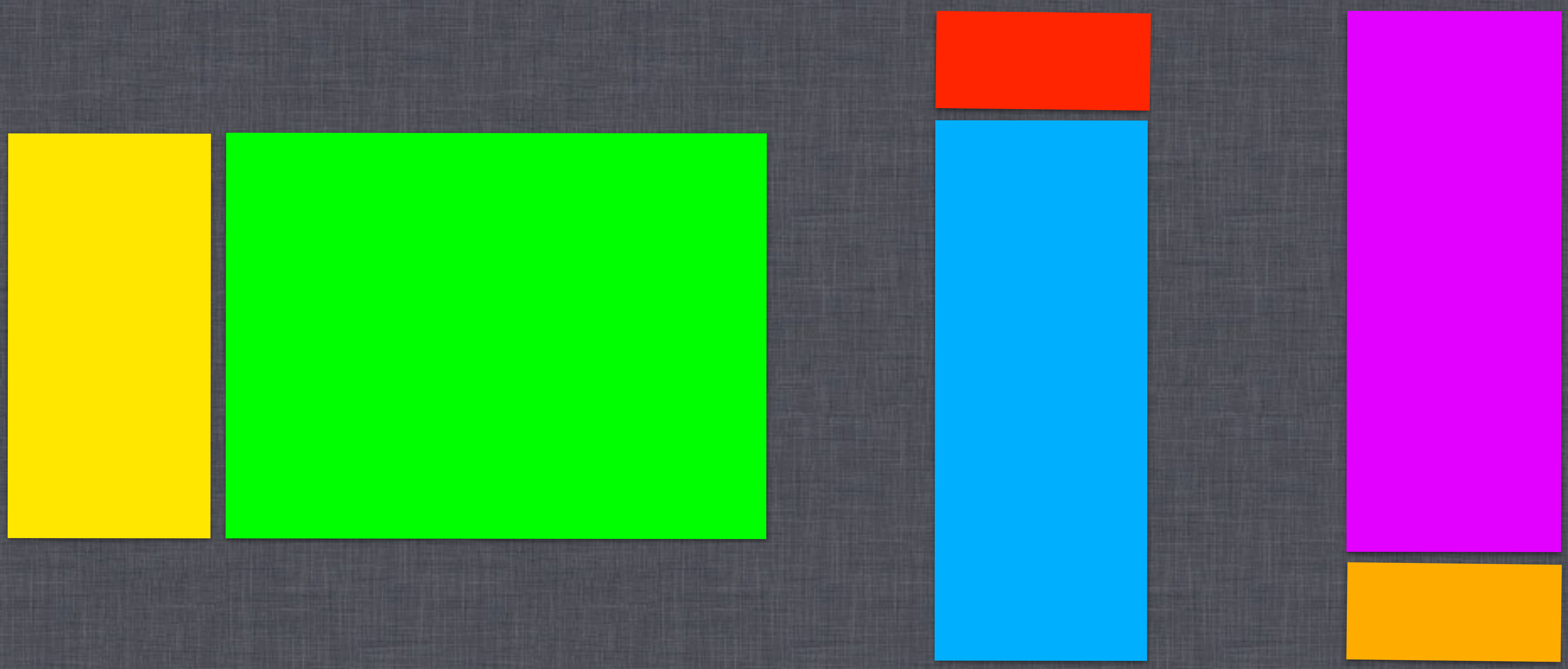
2nd part: Minimal entropy ranking



Small part is the “error”

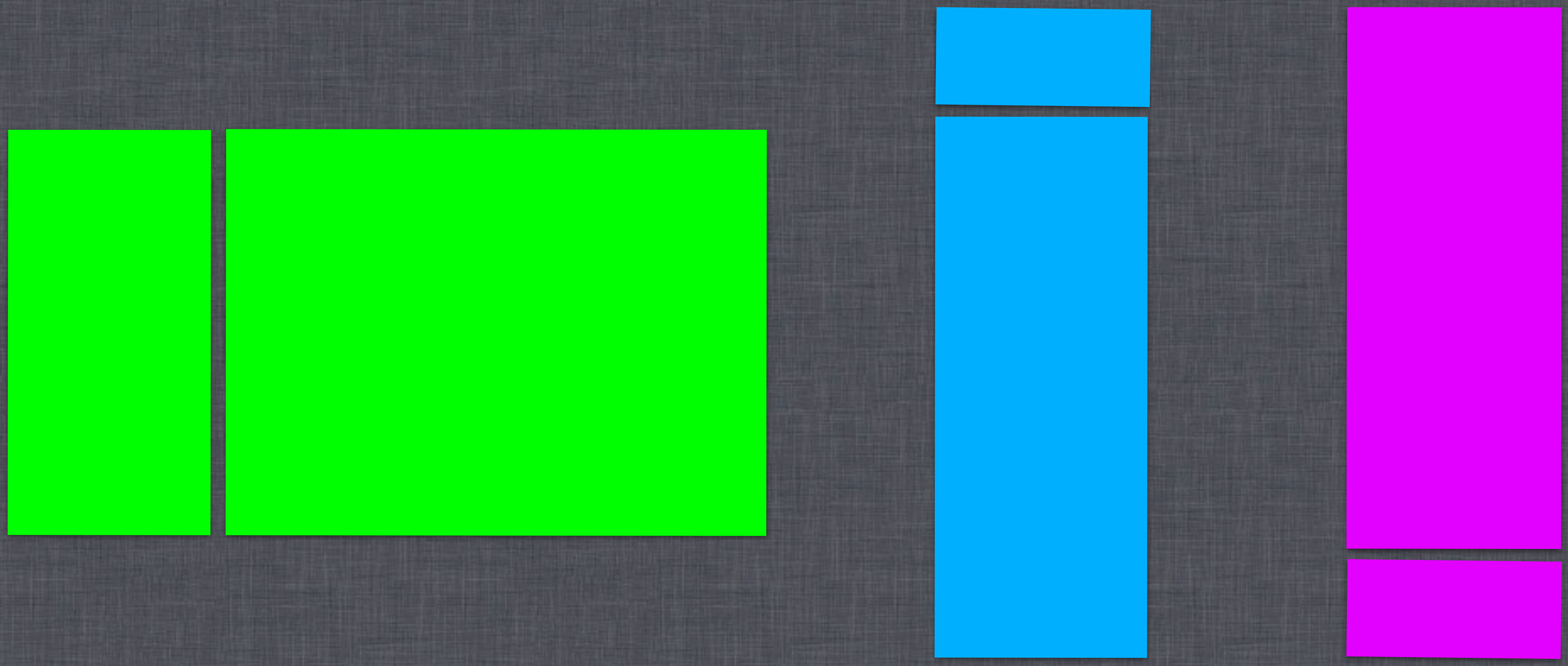
Big part is the “fix”

2nd part: Minimal entropy ranking



"Fixing" an error means
changing the error's fingerprint
so that the whole rectangle maintains one invariant.

2nd part: Minimal entropy ranking



"Fixing" an error means
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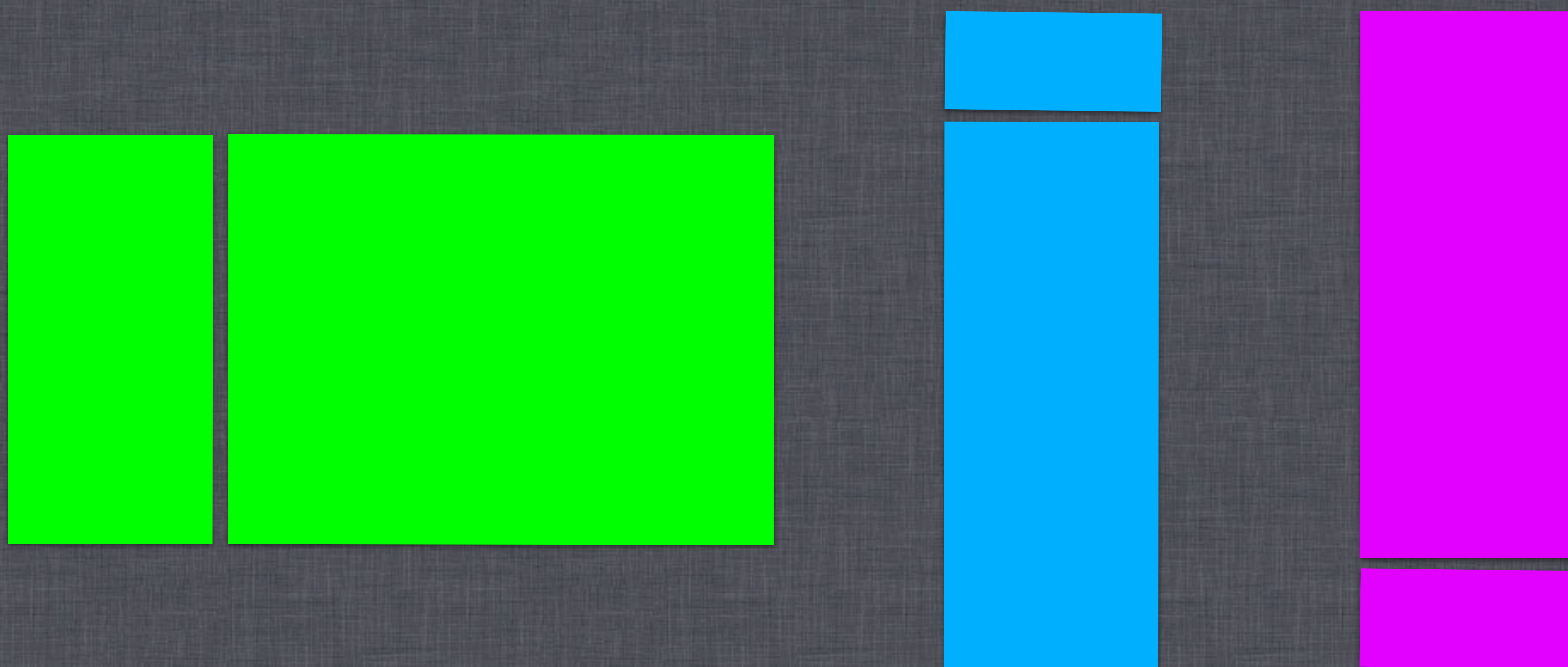
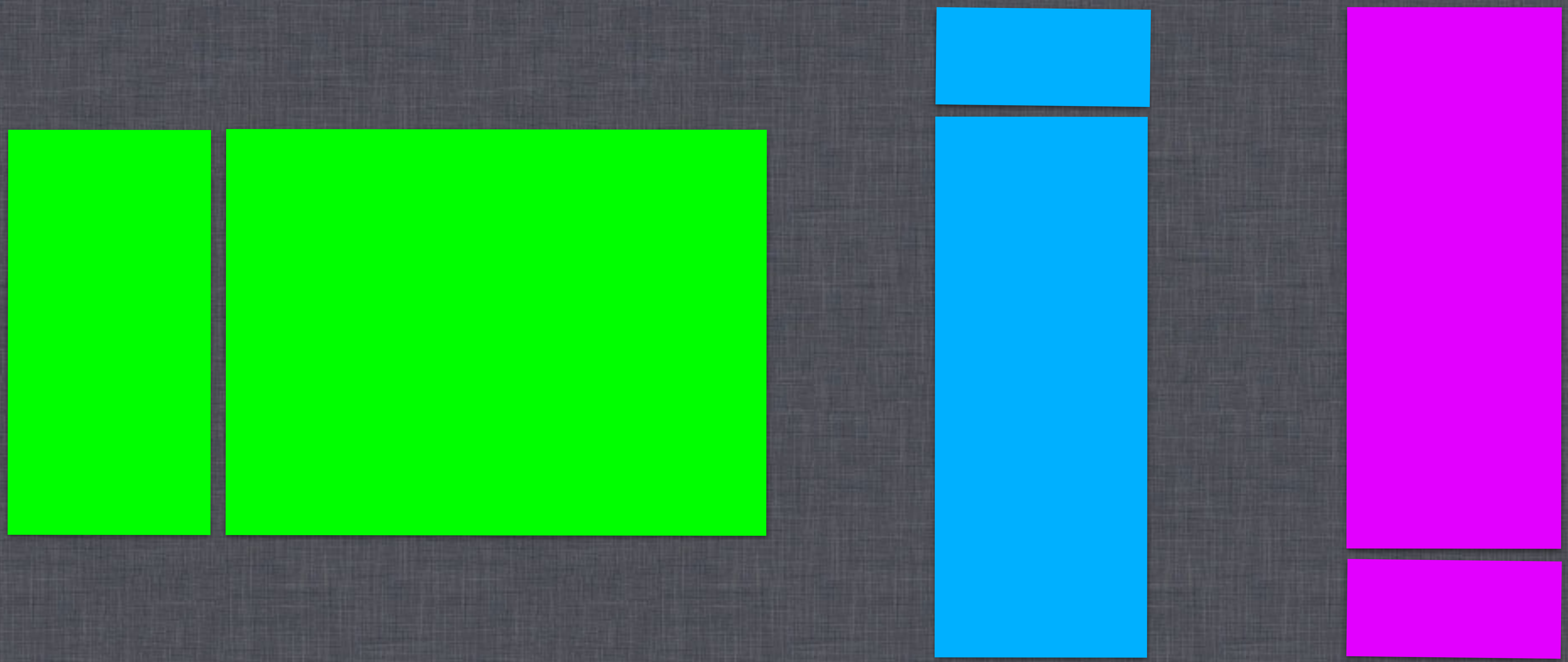
2nd part: Minimal entropy ranking



Rank fixes by three factors.

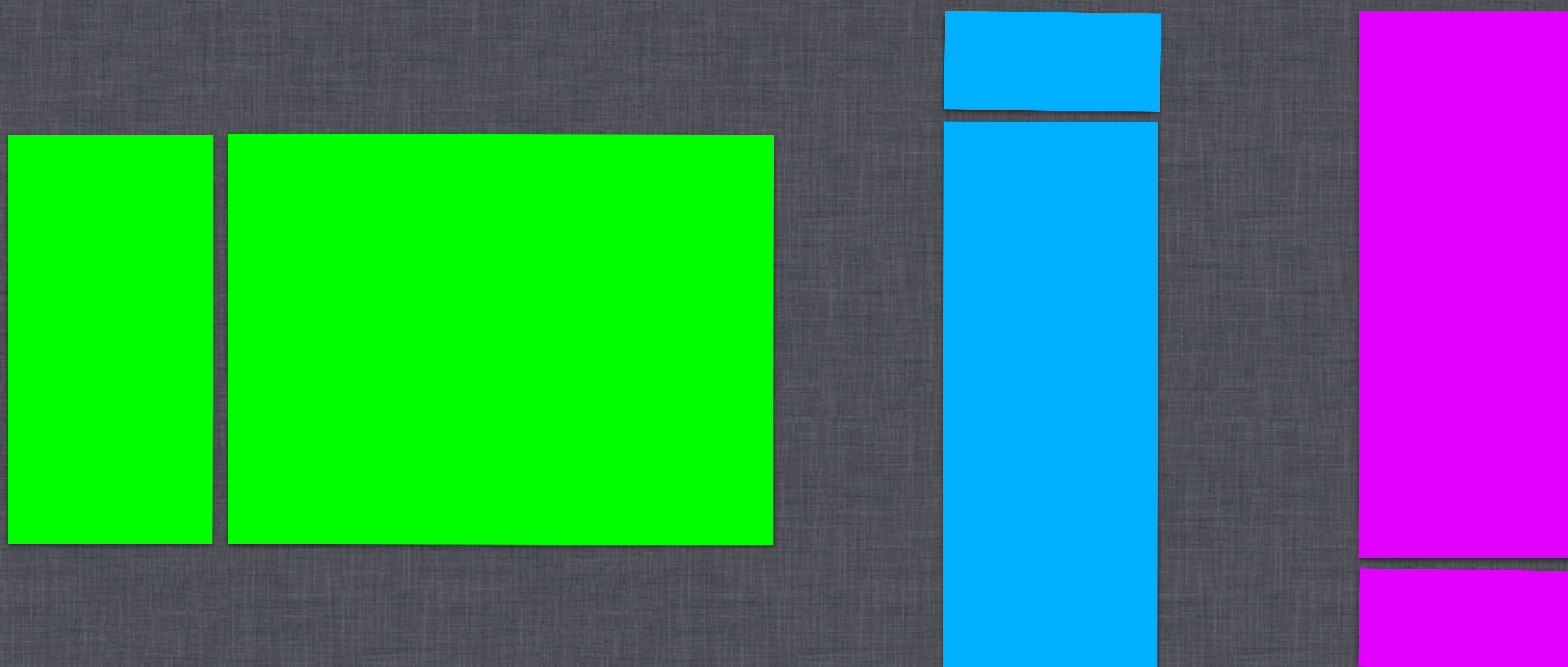
2nd part: Minimal entropy ranking



Rank fixes by three factors.

First: we want fixes that reduce entropy

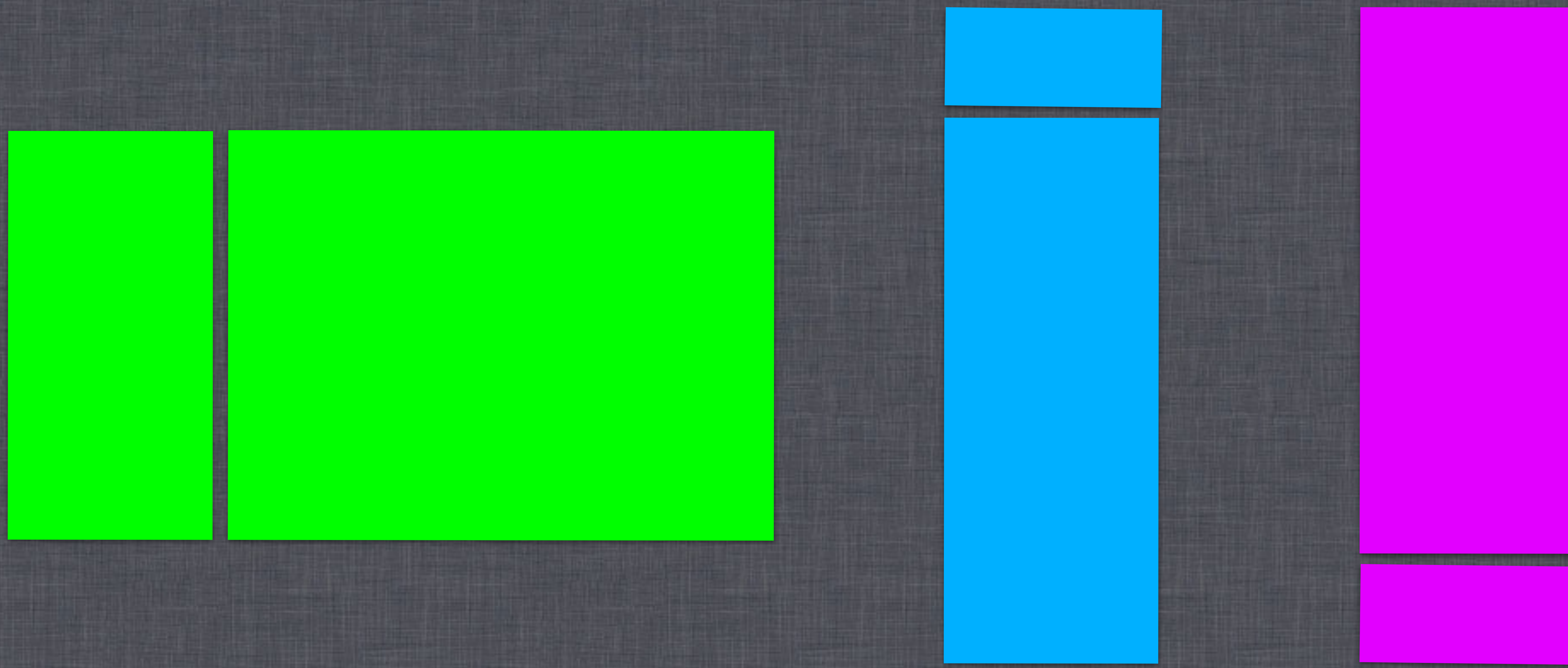
2nd part: Minimal entropy ranking



Rank fixes by three factors.

First: we want fixes that reduce entropy (the least).

2nd part: Minimal entropy ranking

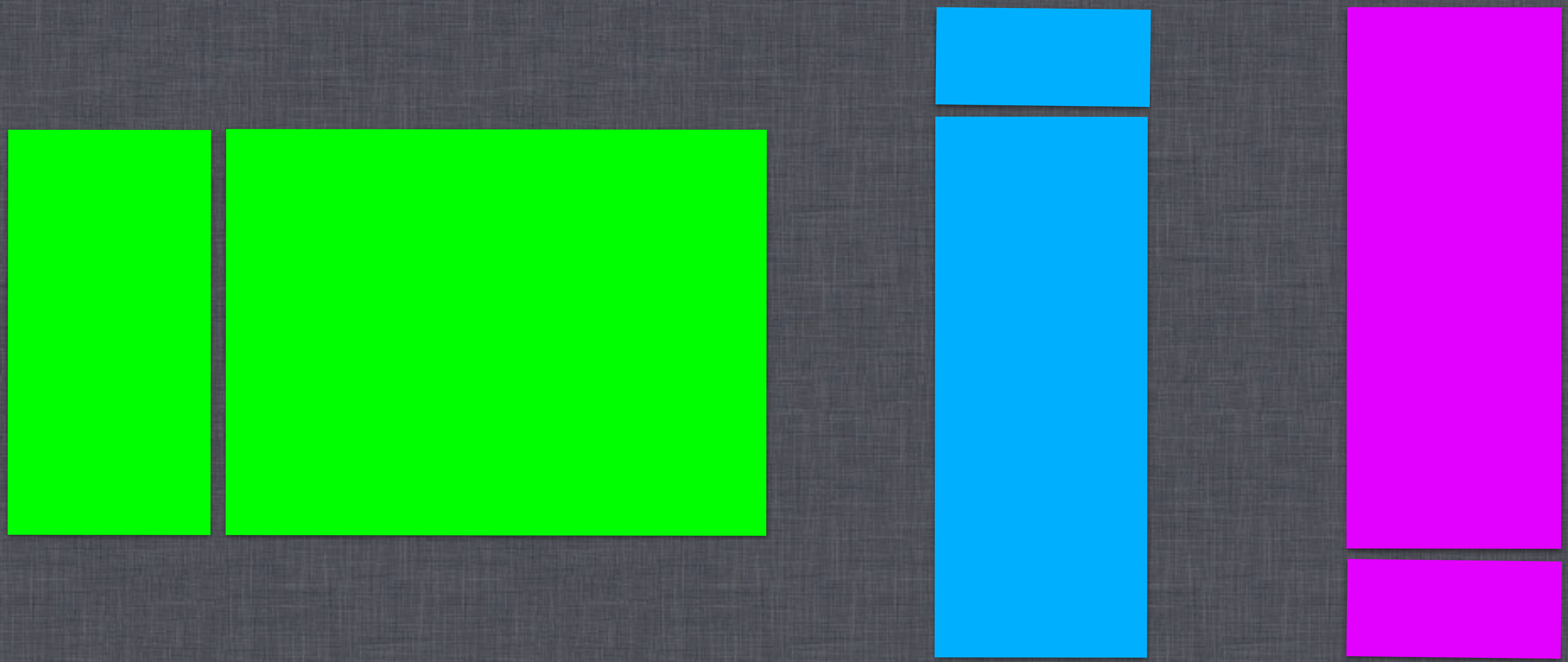


Rank fixes by three factors.

First: we want fixes that reduce entropy (the least).

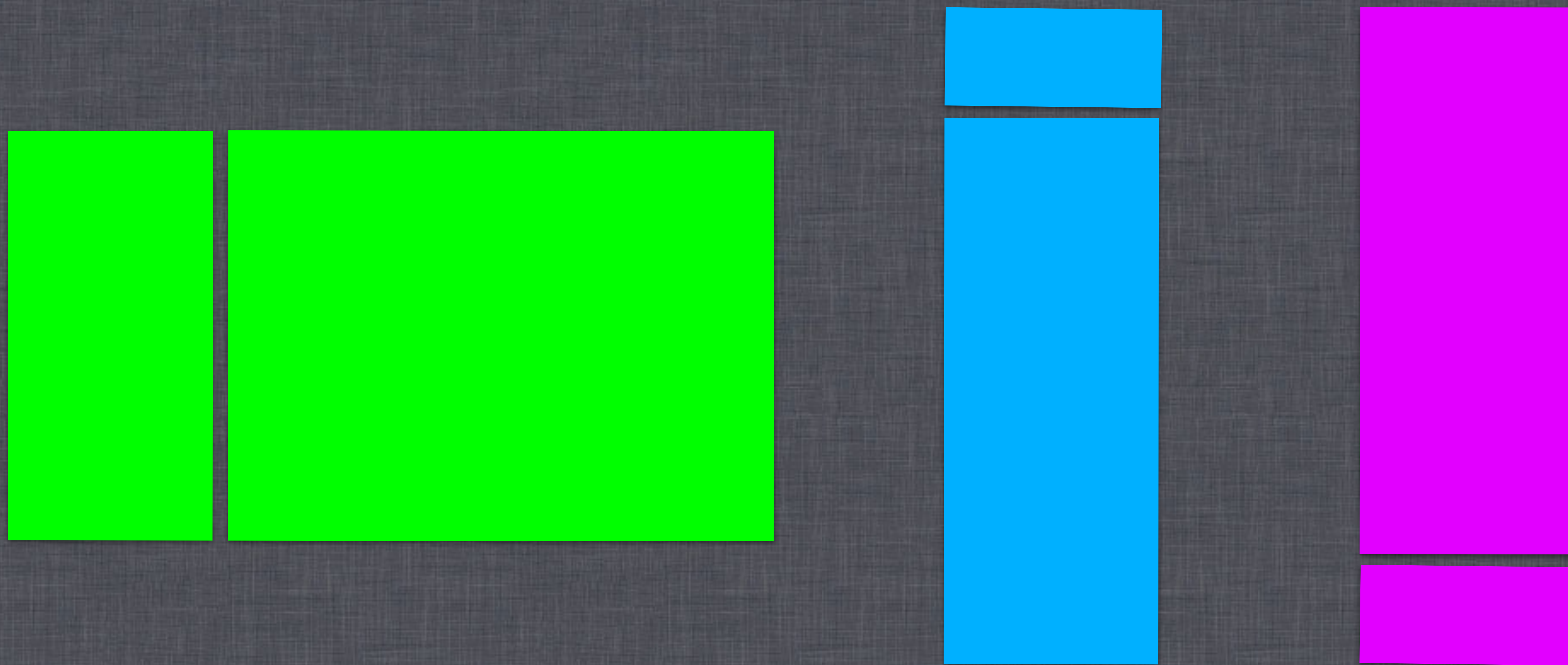
The two on the right are better than the one on the left.

2nd part: Minimal entropy ranking



Second: The distance between fingerprints.

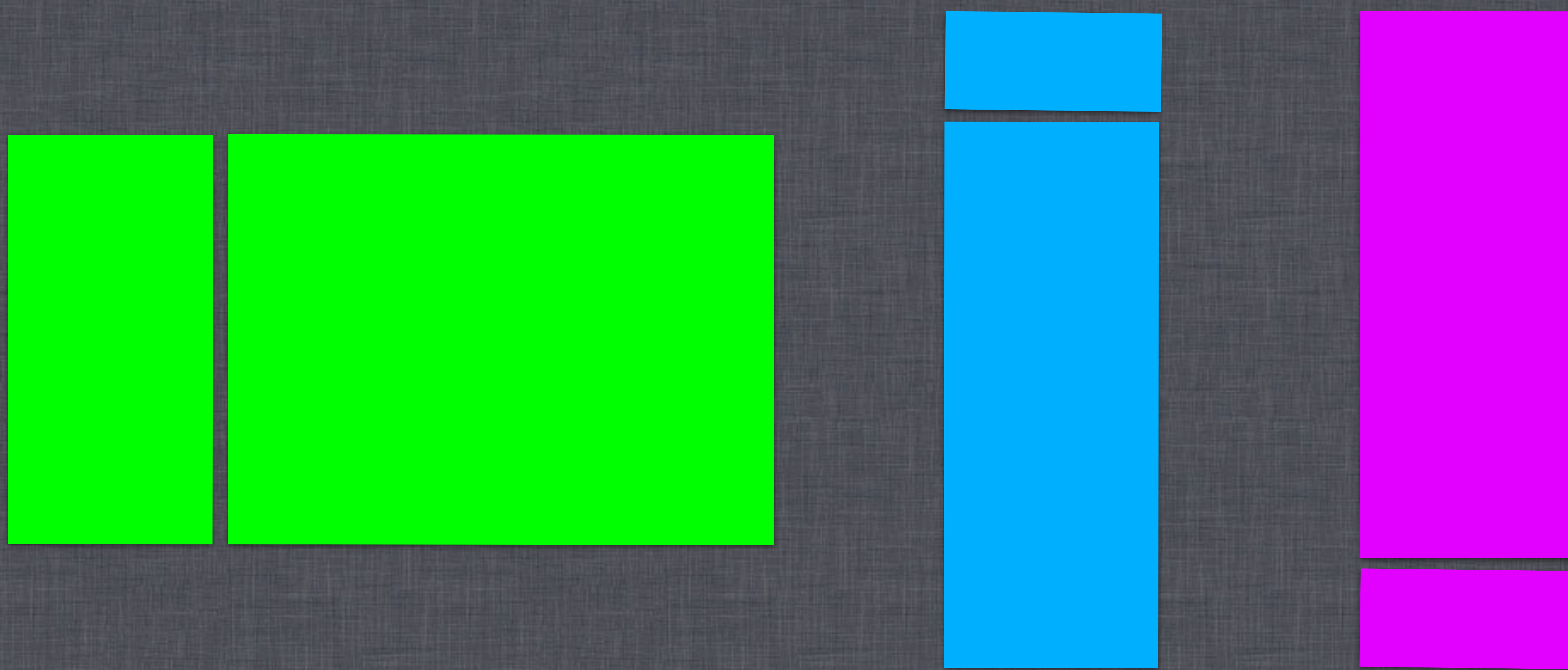
2nd part: Minimal entropy ranking



Second: The distance between fingerprints.

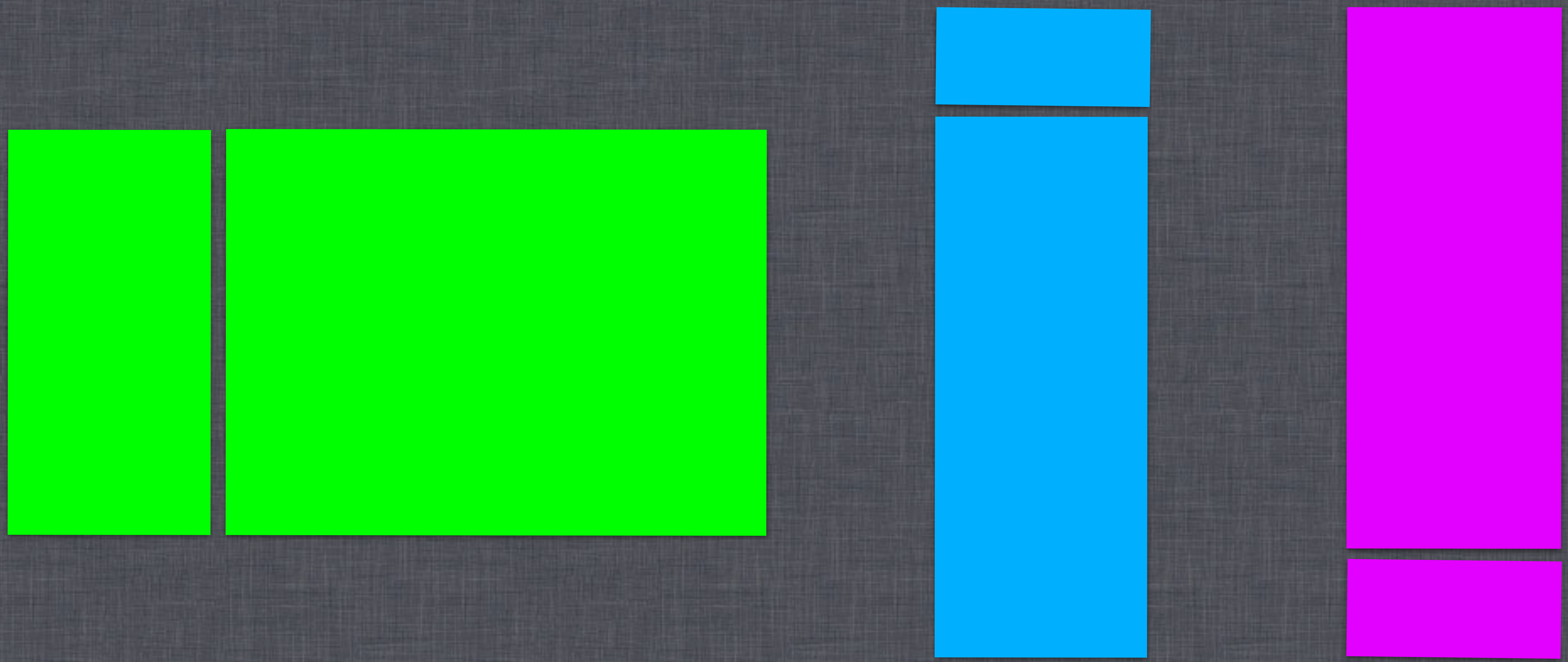
E,g., `=SUM(A1:A10)` is a more probable fix for `=SUM(A1:A9)` than `=MAX(B20:B30)`

2nd part: Minimal entropy ranking



Third: The size of the invariant cluster.

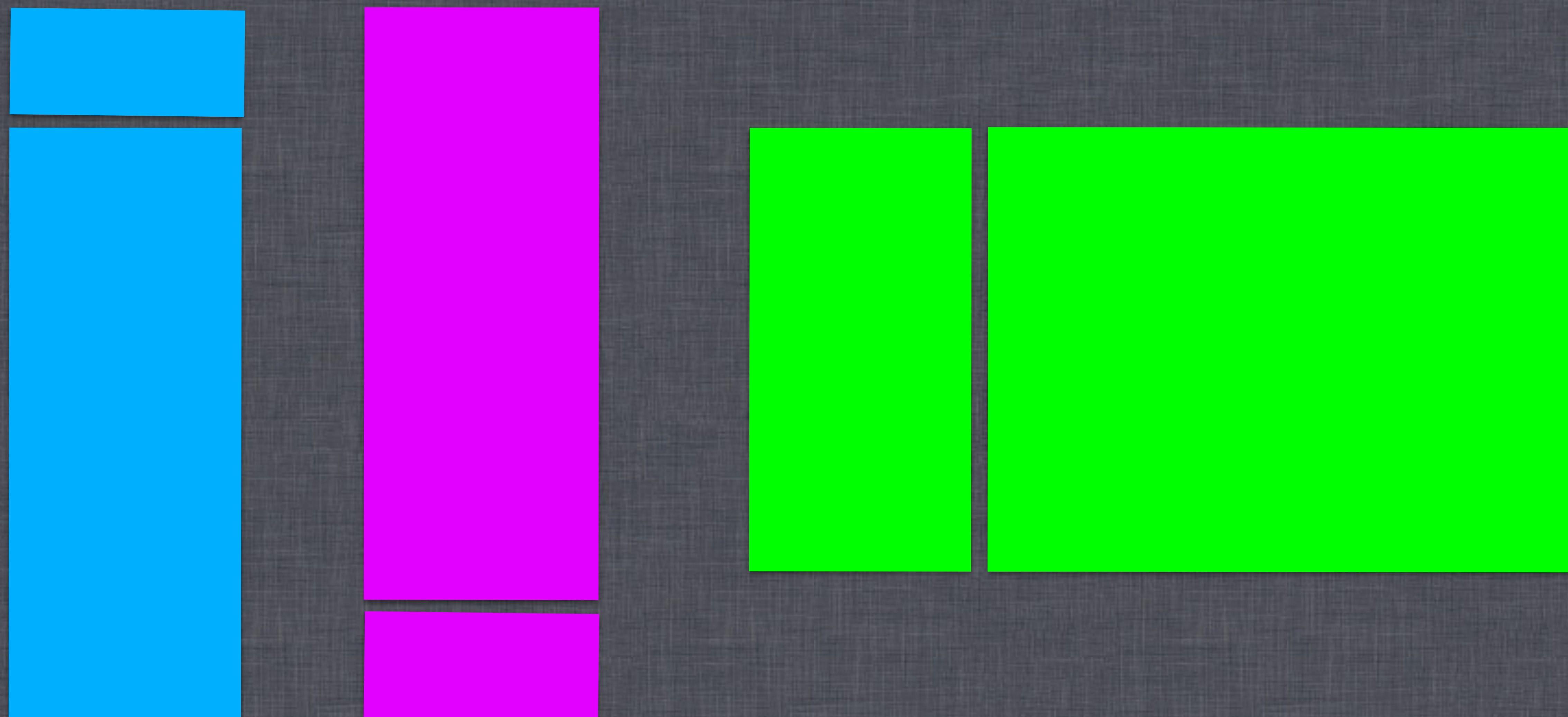
2nd part: Minimal entropy ranking



Third: The size of the invariant cluster.

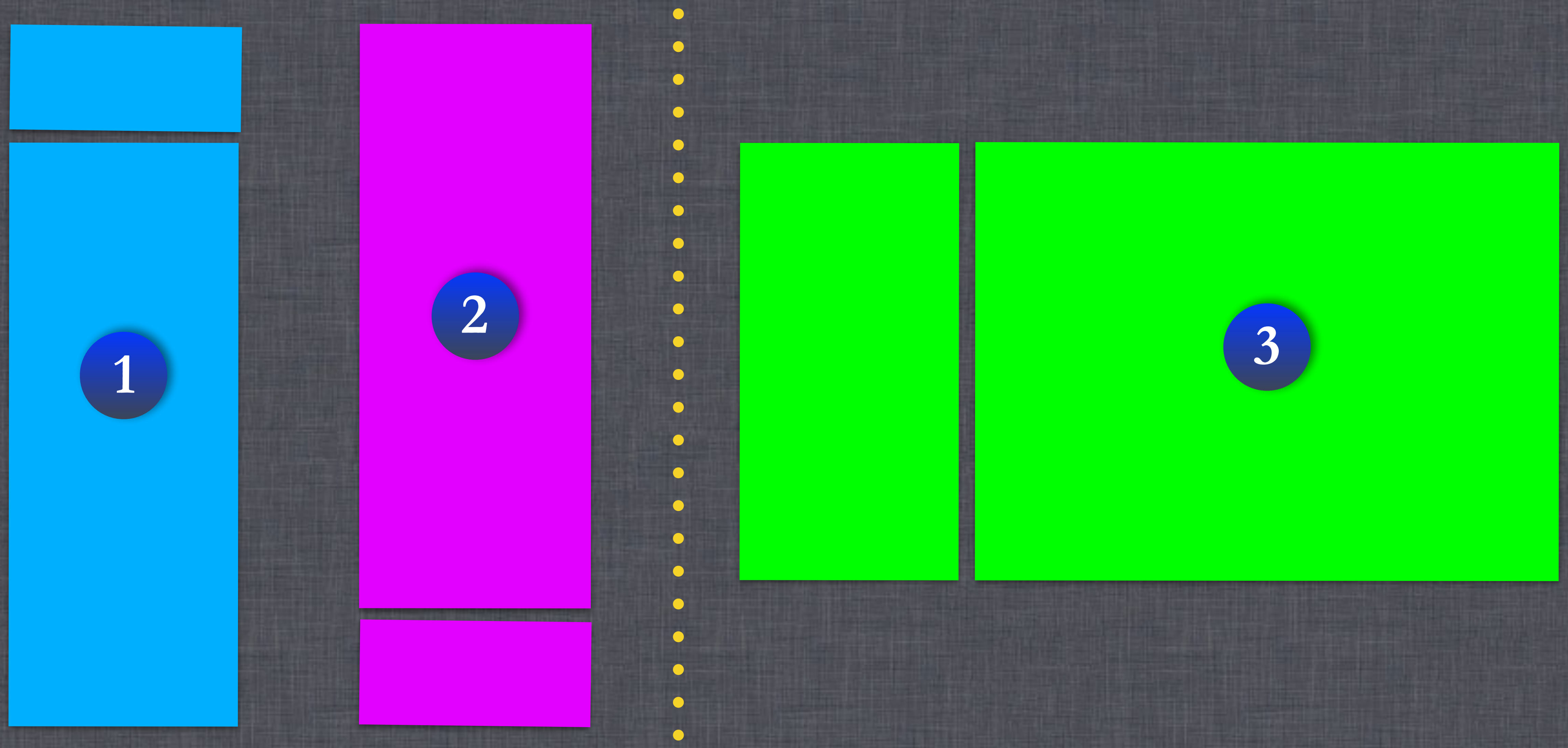
I.e., strongly-held invariants are better candidates than weakly-held invariants.

2nd part: Minimal entropy ranking



Finally, threshold and return ranking.

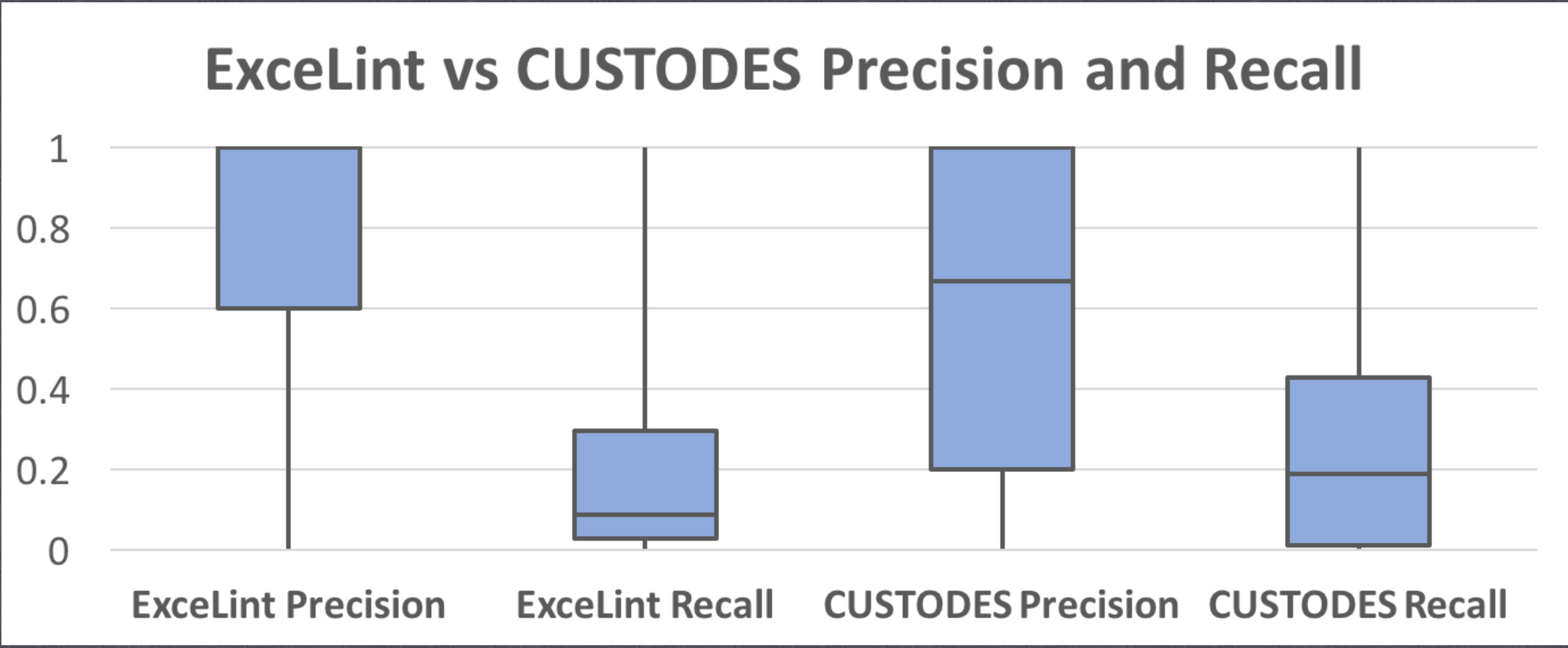
2nd part: Minimal entropy ranking



Finally, threshold and return ranking.

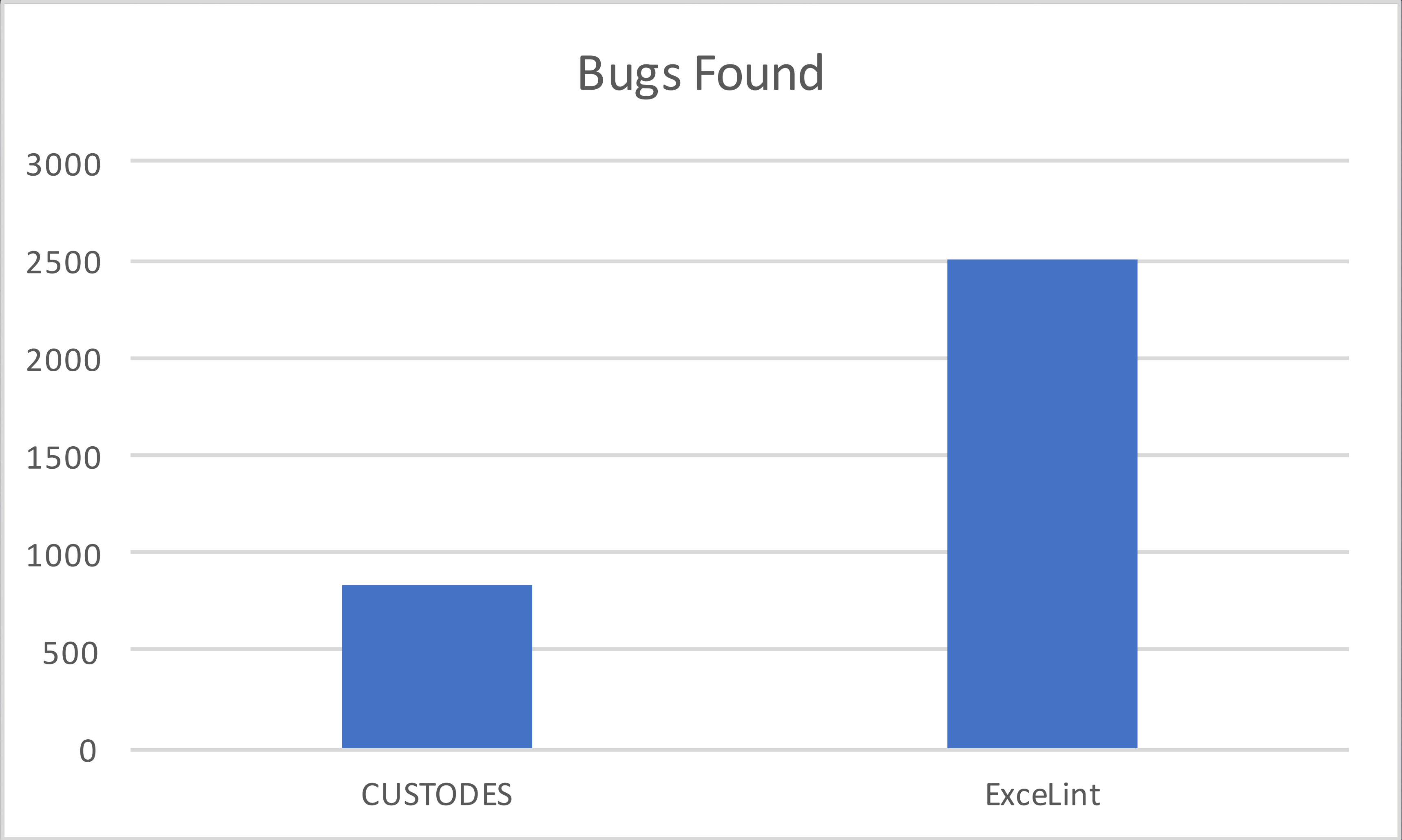
Empirical Results

High precision



Average precision: 76% (vs. 60%)

High *Overall* Recall

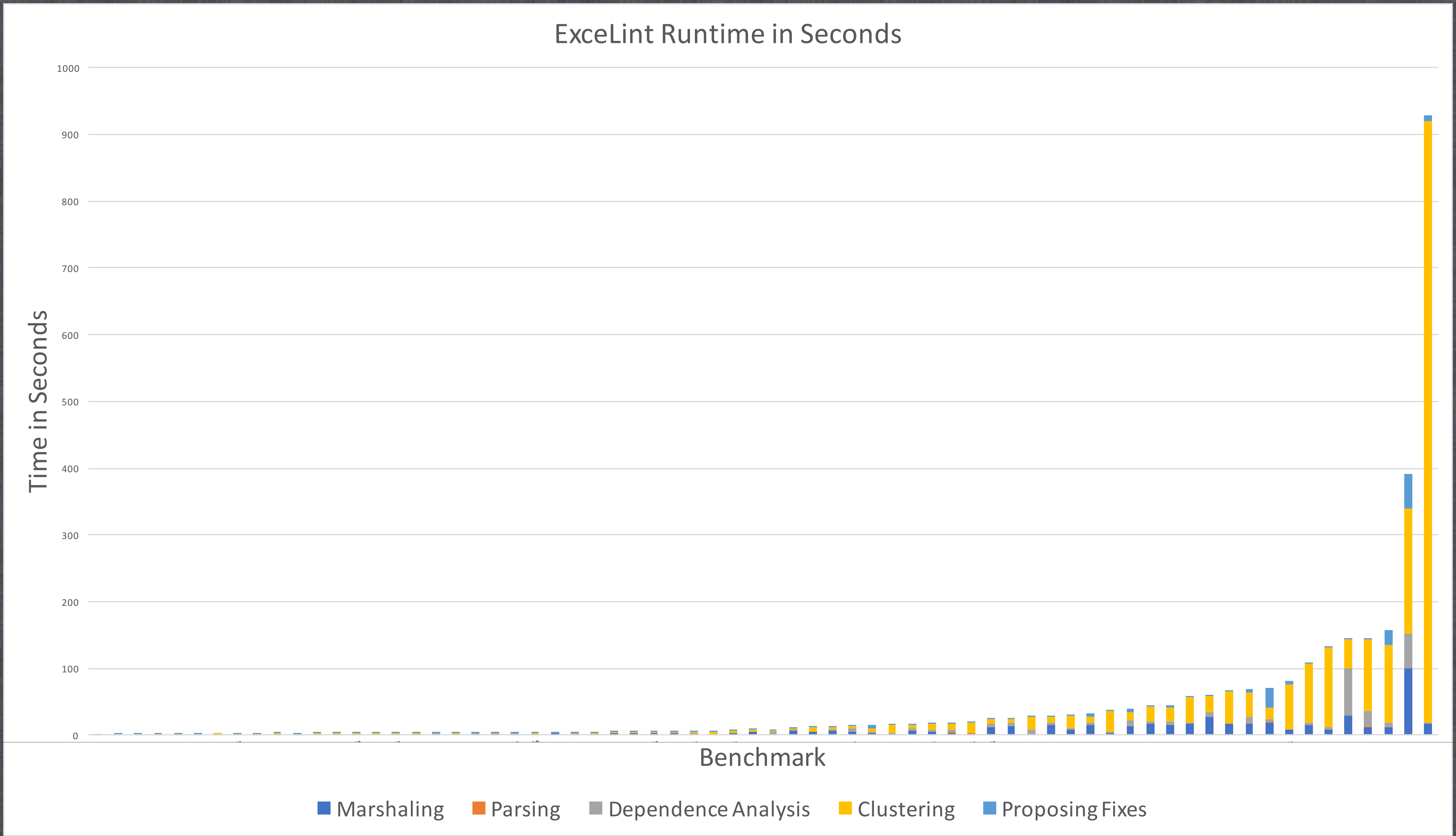


Earlier research ("CUSTODES") found 841 formula errors

Using ExceLint: found 1,658 more

Averaged 2.45 reference bugs per minute

Fast analysis



73% of analyses done under < 30 seconds

Median analysis time: 7.83 seconds

Case study: Reinhart-Rogoff

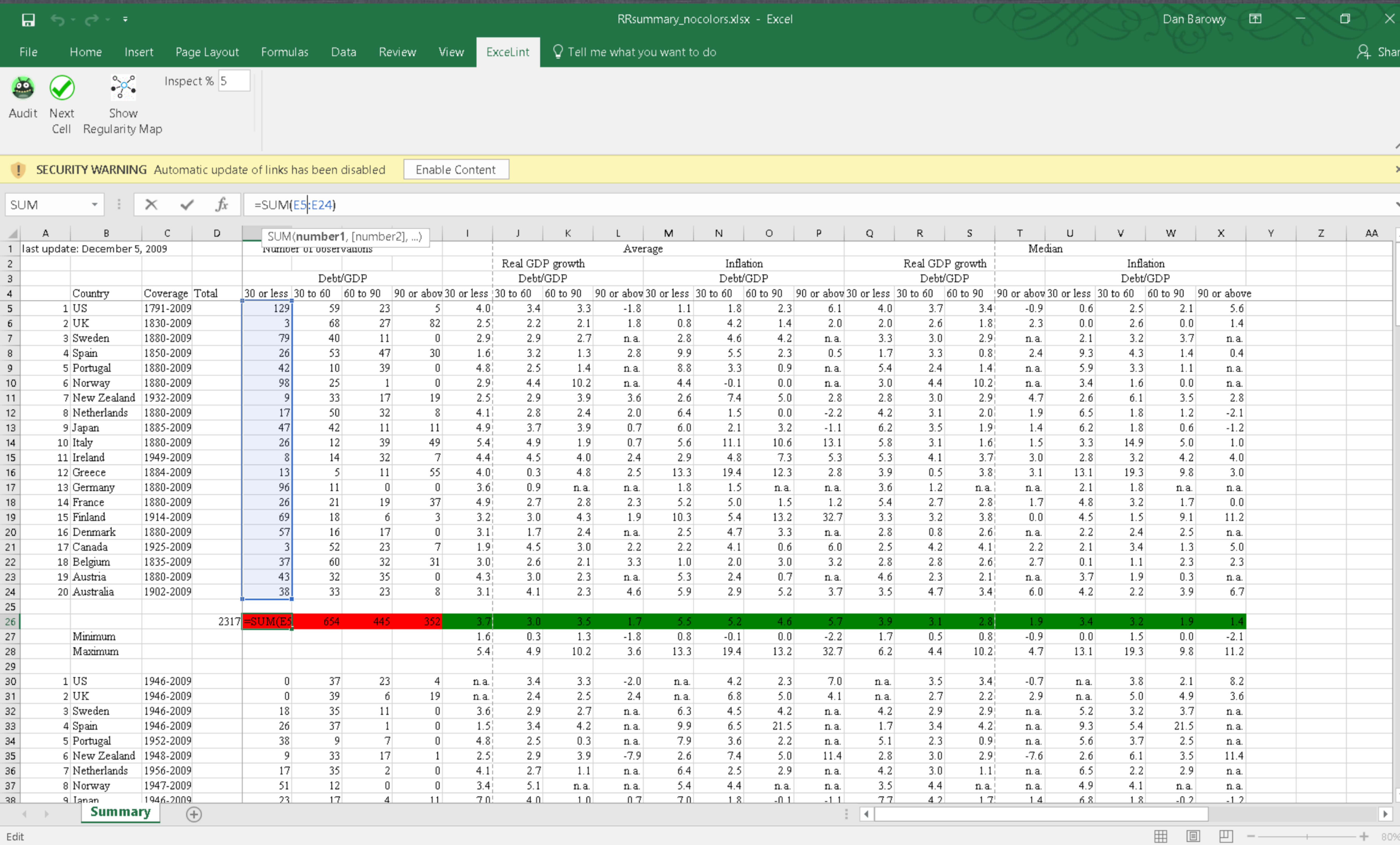
RRsummary_nocolors.xlsx - Excel

E26 =SUM(E5:E24)

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
last update: December 5, 2009				Number of observations						Average									Median				
									Real GDP growth				Inflation				Real GDP growth				Inflation		
					Debt/GDP				Debt/GDP				Debt/GDP				Debt/GDP				Debt/GDP		
	Country	Coverage	Total	30 or less	30 to 60	60 to 90	90 or abov	30 or less	30 to 60	60 to 90	90 or abov	30 or less	30 to 60	60 to 90	90 or abov	30 or less	30 to 60	60 to 90	90 or abov	30 or less	30 to 60	60 to 90	90 or above
1	US	1791-2009		129	59	23	5	4.0	3.4	3.3	-1.8	1.1	1.8	2.3	6.1	4.0	3.7	3.4	-0.9	0.6	2.5	2.1	5.6
2	UK	1830-2009		3	68	27	82	2.5	2.2	2.1	1.8	0.8	4.2	1.4	2.0	2.0	2.6	1.8	2.3	0.0	2.6	0.0	1.4
3	Sweden	1880-2009		79	40	11	0	2.9	2.9	2.7	n.a.	2.8	4.6	4.2	n.a.	3.3	3.0	2.9	n.a.	2.1	3.2	3.7	n.a.
4	Spain	1850-2009		26	53	47	30	1.6	3.2	1.3	2.8	9.9	5.5	2.3	0.5	1.7	3.3	0.8	2.4	9.3	4.3	1.4	0.4
5	Portugal	1880-2009		42	10	39	0	4.8	2.5	1.4	n.a.	8.8	3.3	0.9	n.a.	5.4	2.4	1.4	n.a.	5.9	3.3	1.1	n.a.
6	Norway	1880-2009		98	25	1	0	2.9	4.4	10.2	n.a.	4.4	-0.1	0.0	n.a.	3.0	4.4	10.2	n.a.	3.4	1.6	0.0	n.a.
7	New Zealand	1932-2009		9	33	17	19	2.5	2.9	3.9	3.6	2.6	7.4	5.0	2.8	2.8	3.0	2.9	4.7	2.6	6.1	3.5	2.8
8	Netherlands	1880-2009		17	50	32	8	4.1	2.8	2.4	2.0	6.4	1.5	0.0	-2.2	4.2	3.1	2.0	1.9	6.5	1.8	1.2	-2.1
9	Japan	1885-2009		47	42	11	11	4.9	3.7	3.9	0.7	6.0	2.1	3.2	-1.1	6.2	3.5	1.9	1.4	6.2	1.8	0.6	-1.2
10	Italy	1880-2009		26	12	39	49	5.4	4.9	1.9	0.7	5.6	11.1	10.6	13.1	5.8	3.1	1.6	1.5	3.3	14.9	5.0	1.0
11	Ireland	1949-2009		8	14	32	7	4.4	4.5	4.0	2.4	2.9	4.8	7.3	5.3	5.3	4.1	3.7	3.0	2.8	3.2	4.2	4.0
12	Greece	1884-2009		13	5	11	55	4.0	0.3	4.8	2.5	13.3	19.4	12.3	2.8	3.9	0.5	3.8	3.1	13.1	19.3	9.8	3.0
13	Germany	1880-2009		96	11	0	0	3.6	0.9	n.a.	n.a.	1.8	1.5	n.a.	n.a.	3.6	1.2	n.a.	n.a.	2.1	1.8	n.a.	n.a.
14	France	1880-2009		26	21	19	37	4.9	2.7	2.8	2.3	5.2	5.0	1.5	1.2	5.4	2.7	2.8	1.7	4.8	3.2	1.7	0.0
15	Finland	1914-2009		69	18	6	3	3.2	3.0	4.3	1.9	10.3	5.4	13.2	32.7	3.3	3.2	3.8	0.0	4.5	1.5	9.1	11.2
16	Denmark	1880-2009		57	16	17	0	3.1	1.7	2.4	n.a.	2.5	4.7	3.3	n.a.	2.8	0.8	2.6	n.a.	2.2	2.4	2.5	n.a.
17	Canada	1925-2009		3	52	23	7	1.9	4.5	3.0	2.2	2.2	4.1	0.6	6.0	2.5	4.2	4.1	2.2	2.1	3.4	1.3	5.0
18	Belgium	1835-2009		37	60	32	31	3.0	2.6	2.1	3.3	1.0	2.0	3.0	3.2	2.8	2.8	2.6	2.7	0.1	1.1	2.3	2.3
19	Austria	1880-2009		43	32	35	0	4.3	3.0	2.3	n.a.	5.3	2.4	0.7	n.a.	4.6	2.3	2.1	n.a.	3.7	1.9	0.3	n.a.
20	Australia	1902-2009		38	33	23	8	3.1	4.1	2.3	4.6	5.9	2.9	5.2	3.7	3.5	4.7	3.4	6.0	4.2	2.2	3.9	6.7
			2317	866	654	445	352	3.7	3.0	3.5	1.7	5.5	5.2	4.6	5.7	3.9	3.1	2.8	1.9	3.4	3.2	1.9	1.4
	Minimum							1.6	0.3	1.3	-1.8	0.8	-0.1	0.0	-2.2	1.7	0.5	0.8	-0.9	0.0	1.5	0.0	-2.1
	Maximum							5.4	4.9	10.2	3.6	13.3	19.4	13.2	32.7	6.2	4.4	10.2	4.7	13.1	19.3	9.8	11.2
1	US	1946-2009		0	37	23	4	n.a.	3.4	3.3	-2.0	n.a.	4.2	2.3	7.0	n.a.	3.5	3.4	-0.7	n.a.	3.8	2.1	8.2
2	UK	1946-2009		0	39	6	19	n.a.	2.4	2.5	2.4	n.a.	6.8	5.0	4.1	n.a.	2.7	2.2	2.9	n.a.	5.0	4.9	3.6
3	Sweden	1946-2009		18	35	11	0	3.6	2.9	2.7	n.a.	6.3	4.5	4.2	n.a.	4.2	2.9	2.9	n.a.	5.2	3.2	3.7	n.a.
4	Spain	1946-2009		26	37	1	0	1.5	3.4	4.2	n.a.	9.9	6.5	21.5	n.a.	1.7	3.4	4.2	n.a.	9.3	5.4	21.5	n.a.
5	Portugal	1952-2009		38	9	7	0	4.8	2.5	0.3	n.a.	7.9	3.6	2.2	n.a.	5.1	2.3	0.9	n.a.	5.6	3.7	2.5	n.a.
6	New Zealand	1948-2009		9	33	17	1	2.5	2.9	3.9	-7.9	2.6	7.4	5.0	11.4	2.8	3.0	2.9	-7.6	2.6	6.1	3.5	11.4
7	Netherlands	1956-2009		17	35	2	0	4.1	2.7	1.1	n.a.	6.4	2.5	2.9	n.a.	4.2	3.0	1.1	n.a.	6.5	2.2	2.9	n.a.
8	Norway	1947-2009		51	12	0	0	3.4	5.1	n.a.	n.a.	5.4	4.4	n.a.	n.a.	3.5	4.4	n.a.	n.a.	4.9	4.1	n.a.	n.a.
9	Japan	1946-2009		23	17	4	11	7.0	4.0	1.0	0.7	7.0	1.8	-0.1	-1.1	7.7	4.2	1.7	1.4	6.8	1.8	-0.2	-1.2

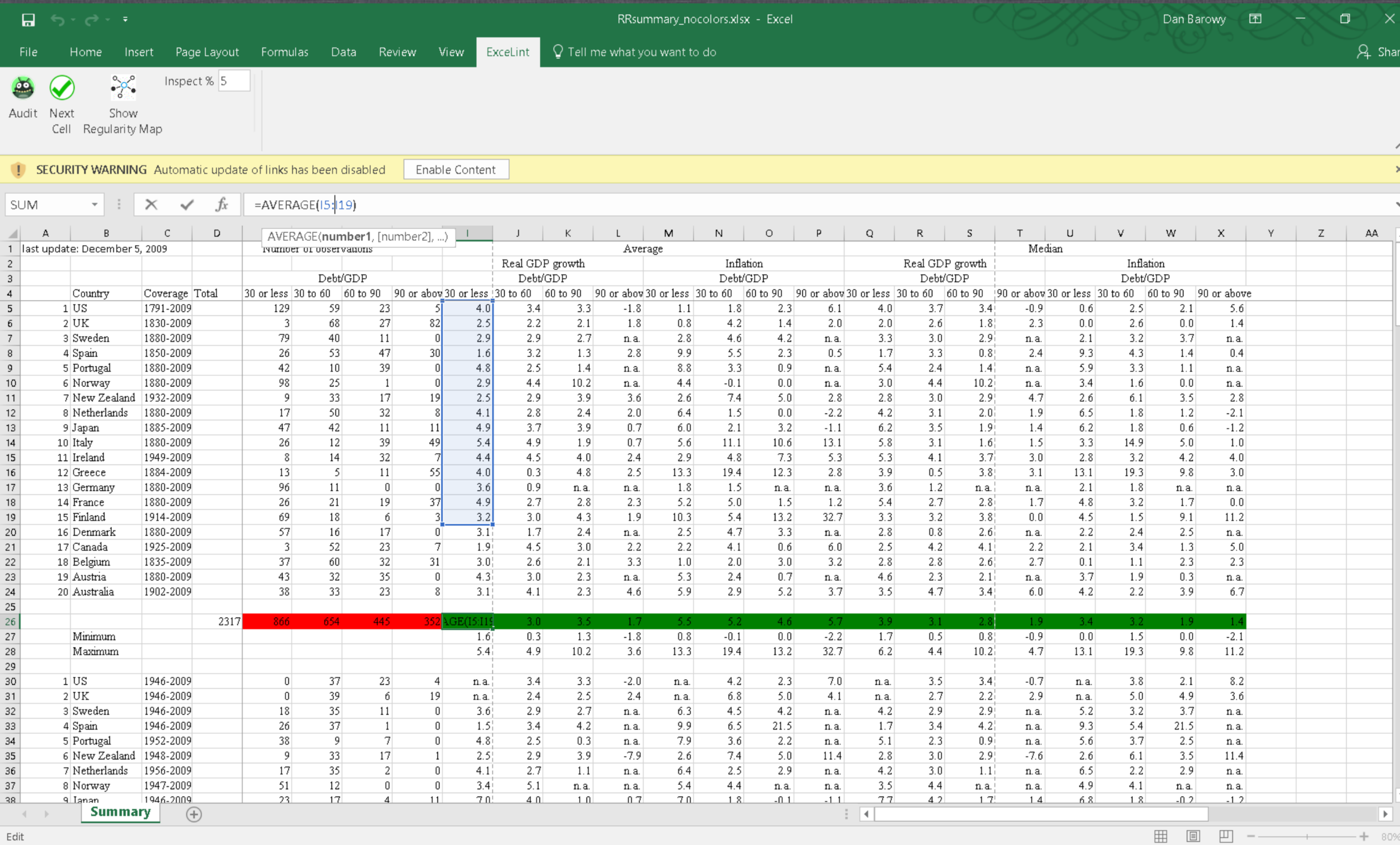
Summary

Case study: Reinhart-Rogoff



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
1	last update: December 5, 2009				Number of observations						Average									Median				
2										Real GDP growth				Inflation				Real GDP growth				Inflation		
3						Debt/GDP				Debt/GDP				Debt/GDP				Debt/GDP				Debt/GDP		
4		Country	Coverage	Total	30 or less	30 to 60	60 to 90	90 or abov	30 or less	30 to 60	60 to 90	90 or abov	30 or less	30 to 60	60 to 90	90 or abov	30 or less	30 to 60	60 to 90	90 or abov	30 or less	30 to 60	60 to 90	90 or above
5	1	US	1791-2009		129	59	23	5	4.0	3.4	3.3	-1.8	1.1	1.8	2.3	6.1	4.0	3.7	3.4	-0.9	0.6	2.5	2.1	5.6
6	2	UK	1830-2009		3	68	27	82	2.5	2.2	2.1	1.8	0.8	4.2	1.4	2.0	2.0	2.6	1.8	2.3	0.0	2.6	0.0	1.4
7	3	Sweden	1880-2009		79	40	11	0	2.9	2.9	2.7	n.a.	2.8	4.6	4.2	n.a.	3.3	3.0	2.9	n.a.	2.1	3.2	3.7	n.a.
8	4	Spain	1850-2009		26	53	47	30	1.6	3.2	1.3	2.8	9.9	5.5	2.3	0.5	1.7	3.3	0.8	2.4	9.3	4.3	1.4	0.4
9	5	Portugal	1880-2009		42	10	39	0	4.8	2.5	1.4	n.a.	8.8	3.3	0.9	n.a.	5.4	2.4	1.4	n.a.	5.9	3.3	1.1	n.a.
10	6	Norway	1880-2009		98	25	1	0	2.9	4.4	10.2	n.a.	4.4	-0.1	0.0	n.a.	3.0	4.4	10.2	n.a.	3.4	1.6	0.0	n.a.
11	7	New Zealand	1932-2009		9	33	17	19	2.5	2.9	3.9	3.6	2.6	7.4	5.0	2.8	2.8	3.0	2.9	4.7	2.6	6.1	3.5	2.8
12	8	Netherlands	1880-2009		17	50	32	8	4.1	2.8	2.4	2.0	6.4	1.5	0.0	-2.2	4.2	3.1	2.0	1.9	6.5	1.8	1.2	-2.1
13	9	Japan	1885-2009		47	42	11	11	4.9	3.7	3.9	0.7	6.0	2.1	3.2	-1.1	6.2	3.5	1.9	1.4	6.2	1.8	0.6	-1.2
14	10	Italy	1880-2009		26	12	39	49	5.4	4.9	1.9	0.7	5.6	11.1	10.6	13.1	5.8	3.1	1.6	1.5	3.3	14.9	5.0	1.0
15	11	Ireland	1949-2009		8	14	32	7	4.4	4.5	4.0	2.4	2.9	4.8	7.3	5.3	5.3	4.1	3.7	3.0	2.8	3.2	4.2	4.0
16	12	Greece	1884-2009		13	5	11	55	4.0	0.3	4.8	2.5	13.3	19.4	12.3	2.8	3.9	0.5	3.8	3.1	13.1	19.3	9.8	3.0
17	13	Germany	1880-2009		96	11	0	0	3.6	0.9	n.a.	n.a.	1.8	1.5	n.a.	n.a.	3.6	1.2	n.a.	n.a.	2.1	1.8	n.a.	n.a.
18	14	France	1880-2009		26	21	19	37	4.9	2.7	2.8	2.3	5.2	5.0	1.5	1.2	5.4	2.7	2.8	1.7	4.8	3.2	1.7	0.0
19	15	Finland	1914-2009		69	18	6	3	3.2	3.0	4.3	1.9	10.3	5.4	13.2	32.7	3.3	3.2	3.8	0.0	4.5	1.5	9.1	11.2
20	16	Denmark	1880-2009		57	16	17	0	3.1	1.7	2.4	n.a.	2.5	4.7	3.3	n.a.	2.8	0.8	2.6	n.a.	2.2	2.4	2.5	n.a.
21	17	Canada	1925-2009		3	52	23	7	1.9	4.5	3.0	2.2	2.2	4.1	0.6	6.0	2.5	4.2	4.1	2.2	2.1	3.4	1.3	5.0
22	18	Belgium	1835-2009		37	60	32	31	3.0	2.6	2.1	3.3	1.0	2.0	3.0	3.2	2.8	2.8	2.6	2.7	0.1	1.1	2.3	2.3
23	19	Austria	1880-2009		43	32	35	0	4.3	3.0	2.3	n.a.	5.3	2.4	0.7	n.a.	4.6	2.3	2.1	n.a.	3.7	1.9	0.3	n.a.
24	20	Australia	1902-2009		38	33	23	8	3.1	4.1	2.3	4.6	5.9	2.9	5.2	3.7	3.5	4.7	3.4	6.0	4.2	2.2	3.9	6.7
25																								
26				2317	=SUM(E5	654	445	352	3.7	3.0	3.5	1.7	5.5	5.2	4.6	5.7	3.9	3.1	2.8	1.9	3.4	3.2	1.9	1.4
27		Minimum							1.6	0.3	1.3	-1.8	0.8	-0.1	0.0	-2.2	1.7	0.5	0.8	-0.9	0.0	1.5	0.0	-2.1
28		Maximum							5.4	4.9	10.2	3.6	13.3	19.4	13.2	32.7	6.2	4.4	10.2	4.7	13.1	19.3	9.8	11.2
29																								
30	1	US	1946-2009		0	37	23	4	n.a.	3.4	3.3	-2.0	n.a.	4.2	2.3	7.0	n.a.	3.5	3.4	-0.7	n.a.	3.8	2.1	8.2
31	2	UK	1946-2009		0	39	6	19	n.a.	2.4	2.5	2.4	n.a.	6.8	5.0	4.1	n.a.	2.7	2.2	2.9	n.a.	5.0	4.9	3.6
32	3	Sweden	1946-2009		18	35	11	0	3.6	2.9	2.7	n.a.	6.3	4.5	4.2	n.a.	4.2	2.9	2.9	n.a.	5.2	3.2	3.7	n.a.
33	4	Spain	1946-2009		26	37	1	0	1.5	3.4	4.2	n.a.	9.9	6.5	21.5	n.a.	1.7	3.4	4.2	n.a.	9.3	5.4	21.5	n.a.
34	5	Portugal	1952-2009		38	9	7	0	4.8	2.5	0.3	n.a.	7.9	3.6	2.2	n.a.	5.1	2.3	0.9	n.a.	5.6	3.7	2.5	n.a.
35	6	New Zealand	1948-2009		9	33	17	1	2.5	2.9	3.9	-7.9	2.6	7.4	5.0	11.4	2.8	3.0	2.9	-7.6	2.6	6.1	3.5	11.4
36	7	Netherlands	1956-2009		17	35	2	0	4.1	2.7	1.1	n.a.	6.4	2.5	2.9	n.a.	4.2	3.0	1.1	n.a.	6.5	2.2	2.9	n.a.
37	8	Norway	1947-2009		51	12	0	0	3.4	5.1	n.a.	n.a.	5.4	4.4	n.a.	n.a.	3.5	4.4	n.a.	n.a.	4.9	4.1	n.a.	n.a.
38	9	Japan	1946-2009		23	17	4	11	7.0	4.0	1.0	0.7	7.0	1.8	-0.1	-1.1	7.7	4.2	1.7	1.4	6.8	1.8	-0.2	-1.2

Case study: Reinhart-Rogoff



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
1	last update: December 5, 2009				Number of observations						Average									Median				
2										Real GDP growth				Inflation				Real GDP growth				Inflation		
3						Debt/GDP				Debt/GDP				Debt/GDP				Debt/GDP				Debt/GDP		
4		Country	Coverage	Total	30 or less	30 to 60	60 to 90	90 or abov	30 or less	30 to 60	60 to 90	90 or abov	30 or less	30 to 60	60 to 90	90 or abov	30 or less	30 to 60	60 to 90	90 or abov	30 or less	30 to 60	60 to 90	90 or above
5	1	US	1791-2009		129	59	23	5	4.0	3.4	3.3	-1.8	1.1	1.8	2.3	6.1	4.0	3.7	3.4	-0.9	0.6	2.5	2.1	5.6
6	2	UK	1830-2009		3	68	27	82	2.5	2.2	2.1	1.8	0.8	4.2	1.4	2.0	2.0	2.6	1.8	2.3	0.0	2.6	0.0	1.4
7	3	Sweden	1880-2009		79	40	11	0	2.9	2.9	2.7	n.a.	2.8	4.6	4.2	n.a.	3.3	3.0	2.9	n.a.	2.1	3.2	3.7	n.a.
8	4	Spain	1850-2009		26	53	47	30	1.6	3.2	1.3	2.8	9.9	5.5	2.3	0.5	1.7	3.3	0.8	2.4	9.3	4.3	1.4	0.4
9	5	Portugal	1880-2009		42	10	39	0	4.8	2.5	1.4	n.a.	8.8	3.3	0.9	n.a.	5.4	2.4	1.4	n.a.	5.9	3.3	1.1	n.a.
10	6	Norway	1880-2009		98	25	1	0	2.9	4.4	10.2	n.a.	4.4	-0.1	0.0	n.a.	3.0	4.4	10.2	n.a.	3.4	1.6	0.0	n.a.
11	7	New Zealand	1932-2009		9	33	17	19	2.5	2.9	3.9	3.6	2.6	7.4	5.0	2.8	2.8	3.0	2.9	4.7	2.6	6.1	3.5	2.8
12	8	Netherlands	1880-2009		17	50	32	8	4.1	2.8	2.4	2.0	6.4	1.5	0.0	-2.2	4.2	3.1	2.0	1.9	6.5	1.8	1.2	-2.1
13	9	Japan	1885-2009		47	42	11	11	4.9	3.7	3.9	0.7	6.0	2.1	3.2	-1.1	6.2	3.5	1.9	1.4	6.2	1.8	0.6	-1.2
14	10	Italy	1880-2009		26	12	39	49	5.4	4.9	1.9	0.7	5.6	11.1	10.6	13.1	5.8	3.1	1.6	1.5	3.3	14.9	5.0	1.0
15	11	Ireland	1949-2009		8	14	32	7	4.4	4.5	4.0	2.4	2.9	4.8	7.3	5.3	5.3	4.1	3.7	3.0	2.8	3.2	4.2	4.0
16	12	Greece	1884-2009		13	5	11	55	4.0	0.3	4.8	2.5	13.3	19.4	12.3	2.8	3.9	0.5	3.8	3.1	13.1	19.3	9.8	3.0
17	13	Germany	1880-2009		96	11	0	0	3.6	0.9	n.a.	n.a.	1.8	1.5	n.a.	n.a.	3.6	1.2	n.a.	n.a.	2.1	1.8	n.a.	n.a.
18	14	France	1880-2009		26	21	19	37	4.9	2.7	2.8	2.3	5.2	5.0	1.5	1.2	5.4	2.7	2.8	1.7	4.8	3.2	1.7	0.0
19	15	Finland	1914-2009		69	18	6	3	3.2	3.0	4.3	1.9	10.3	5.4	13.2	32.7	3.3	3.2	3.8	0.0	4.5	1.5	9.1	11.2
20	16	Denmark	1880-2009		57	16	17	0	3.1	1.7	2.4	n.a.	2.5	4.7	3.3	n.a.	2.8	0.8	2.6	n.a.	2.2	2.4	2.5	n.a.
21	17	Canada	1925-2009		3	52	23	7	1.9	4.5	3.0	2.2	2.2	4.1	0.6	6.0	2.5	4.2	4.1	2.2	2.1	3.4	1.3	5.0
22	18	Belgium	1835-2009		37	60	32	31	3.0	2.6	2.1	3.3	1.0	2.0	3.0	3.2	2.8	2.8	2.6	2.7	0.1	1.1	2.3	2.3
23	19	Austria	1880-2009		43	32	35	0	4.3	3.0	2.3	n.a.	5.3	2.4	0.7	n.a.	4.6	2.3	2.1	n.a.	3.7	1.9	0.3	n.a.
24	20	Australia	1902-2009		38	33	23	8	3.1	4.1	2.3	4.6	5.9	2.9	5.2	3.7	3.5	4.7	3.4	6.0	4.2	2.2	3.9	6.7
25																								
26				2317	866	654	445	352	AGE(I5:I19	3.0	3.5	1.7	5.5	5.2	4.6	5.7	3.9	3.1	2.8	1.9	3.4	3.2	1.9	1.4
27		Minimum							1.6	0.3	1.3	-1.8	0.8	-0.1	0.0	-2.2	1.7	0.5	0.8	-0.9	0.0	1.5	0.0	-2.1
28		Maximum							5.4	4.9	10.2	3.6	13.3	19.4	13.2	32.7	6.2	4.4	10.2	4.7	13.1	19.3	9.8	11.2
29																								
30	1	US	1946-2009		0	37	23	4	n.a.	3.4	3.3	-2.0	n.a.	4.2	2.3	7.0	n.a.	3.5	3.4	-0.7	n.a.	3.8	2.1	8.2
31	2	UK	1946-2009		0	39	6	19	n.a.	2.4	2.5	2.4	n.a.	6.8	5.0	4.1	n.a.	2.7	2.2	2.9	n.a.	5.0	4.9	3.6
32	3	Sweden	1946-2009		18	35	11	0	3.6	2.9	2.7	n.a.	6.3	4.5	4.2	n.a.	4.2	2.9	2.9	n.a.	5.2	3.2	3.7	n.a.
33	4	Spain	1946-2009		26	37	1	0	1.5	3.4	4.2	n.a.	9.9	6.5	21.5	n.a.	1.7	3.4	4.2	n.a.	9.3	5.4	21.5	n.a.
34	5	Portugal	1952-2009		38	9	7	0	4.8	2.5	0.3	n.a.	7.9	3.6	2.2	n.a.	5.1	2.3	0.9	n.a.	5.6	3.7	2.5	n.a.
35	6	New Zealand	1948-2009		9	33	17	1	2.5	2.9	3.9	-7.9	2.6	7.4	5.0	11.4	2.8	3.0	2.9	-7.6	2.6	6.1	3.5	11.4
36	7	Netherlands	1956-2009		17	35	2	0	4.1	2.7	1.1	n.a.	6.4	2.5	2.9	n.a.	4.2	3.0	1.1	n.a.	6.5	2.2	2.9	n.a.
37	8	Norway	1947-2009		51	12	0	0	3.4	5.1	n.a.	n.a.	5.4	4.4	n.a.	n.a.	3.5	4.4	n.a.	n.a.	4.9	4.1	n.a.	n.a.
38	9	Japan	1946-2009		23	17	4	11	7.0	4.0	1.0	0.7	7.0	1.8	-0.1	-1.1	7.7	4.2	1.7	1.4	6.8	1.8	-0.2	-1.2

ExceLint

Introduces *geospatial static analysis*

to find formula errors (& fixes)

with high precision

	D	E	F	G
5	*Week3*	*Week 4*	*Total Hours*	*Overtime Hrs*
6	5.25	8.58	34.33	0.00
7	20.50	17.83	53.50	0.00
8	16.00	16.83	50.50	0.00
9	43.00	41.17	123.50	5.50
10	48.00	44.00	132.00	12.00
11	38.50	35.50	106.50	5.00

	E	F	G
5	*Week 4*	*Total Hours*	*Overtime Hrs*
6	8.58	34.33	0.00
7	17.83	53.50	0.00
8	16.83	50.50	0.00
9	41.17	123.50	5.50
10	44.00	132.00	12.00
11	35.50	106.50	5.00

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