Randomized Benchmarking and Clifford Group

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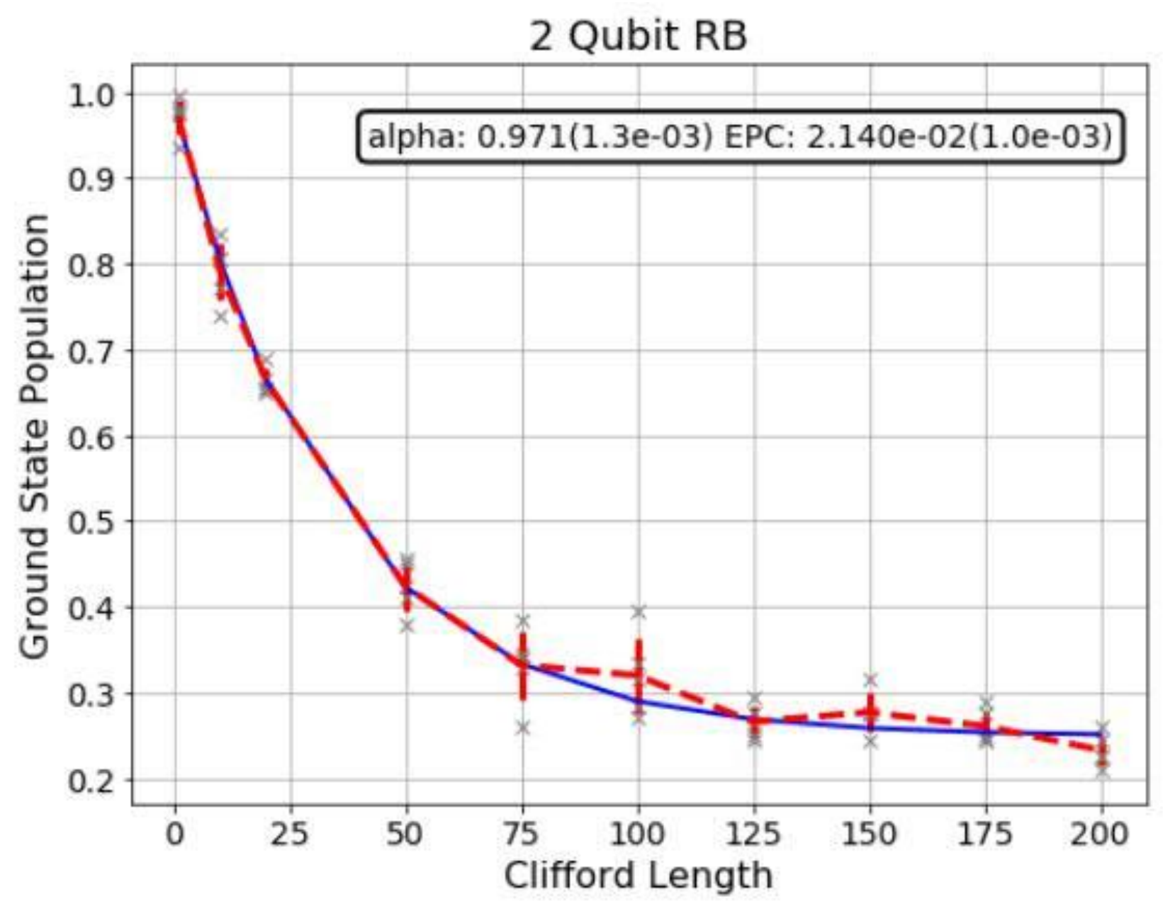
Randomized Benchmarking (RB)

- A proven protocol that provides an efficient and reliable estimate of an average error-rate for a set of quantum gate operations [13,14,24,27]
- Consists of the following three stages:

a) Generate RB sequences consisting of random elements from a Clifford group, including a computed reversal gate

b) Run the RB sequences either on the device or on a simulator (with a noise model) and compare to the initial state

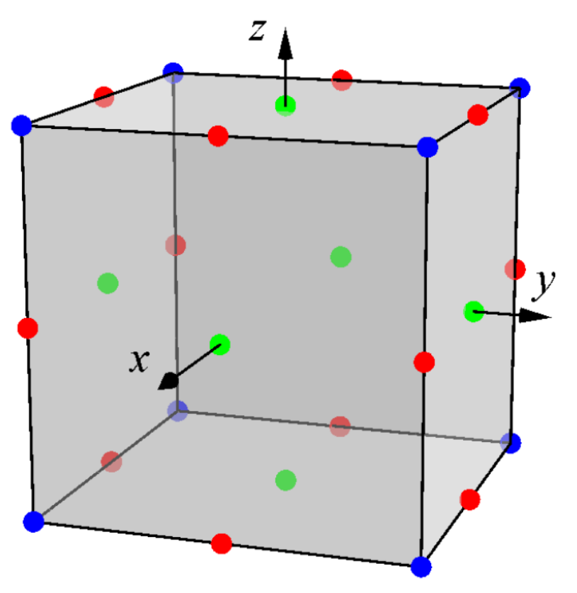
c) Get the statistics and fit an exponential decaying curve: Ap^m+B
Error per Clifford (EPC): $r=(1-p)(d-1)/d$
$(d=2^n)$



Clifford Group

- The **Clifford group** consists of the quantum operators that can be efficiently simulated (in polynomial time) using a classical computer – Clifford simulation [1,10]
- It is generated by the gates: *H*, *S* and *CNOT:*

$$H=\frac{1}{\sqrt{2}}\begin{pmatrix}1&1\\1&-1\end{pmatrix},S=\begin{pmatrix}1&0\\0&i\end{pmatrix},CNOT=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}$$

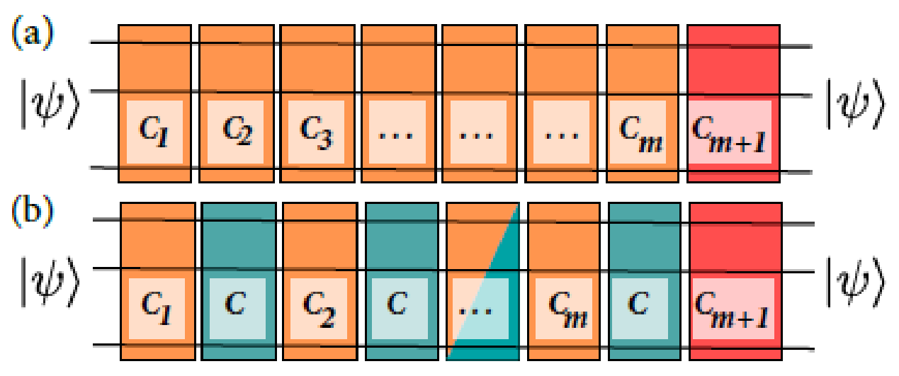


- The Clifford group on 1-qubit has 24 elements (the rotational symmetries of the cube)
- The Clifford group on n-qubits is of size: $2^{n^2+2n}\prod_{j=1}^{n}(4^j-1)$
- Efficient algorithms to generate and synthesize Clifford elements [2,7,11,12,23]
- The Clifford group forms a *unitary 2-design* –
twirling the finite Clifford group is equivalent to *twirling* the infinite unitary group

RB methods for estimating noise parameters

- **Interleaved RB** [15,25,28]**:** Estimating the average error of individual quantum gates

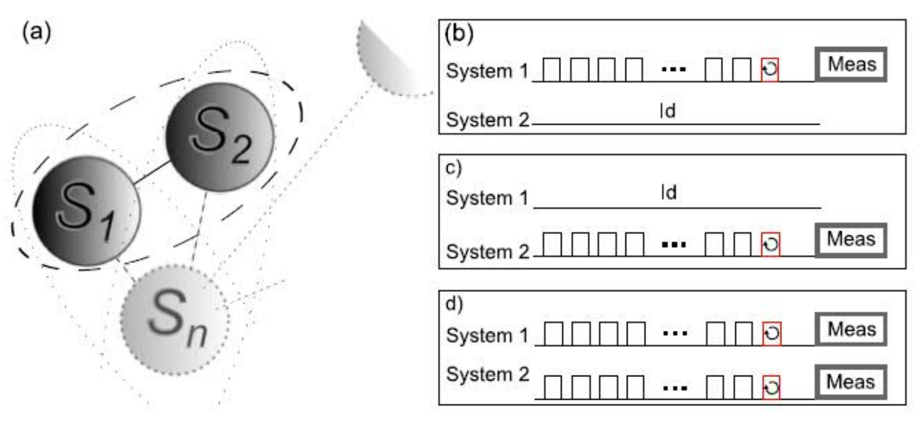
$$r_{\mathcal{C}}^{\text{est}}=\frac{(d-1)\,(1-p_{\overline{\mathcal{C}}}/p)}{d},$$



- **Purity RB** [16,18,26,28]**:** Quantifies how coherent the errors are, namely

$$Tr(\rho^2)=1 \Leftrightarrow \rho \text{ is pure (coherent)}$$

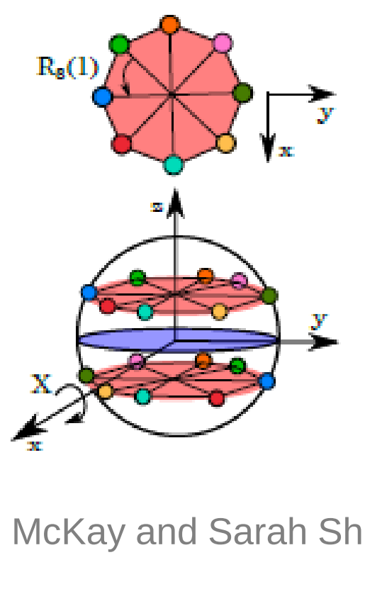
- **Leakage RB** [19]**:** Quantification and characterization of leakage errors (unwanted energy levels)
- **Simultaneous RB** [8]**:** Running RB simultaneously on subsets of qubits, characterizing the amount of addressability between subsystems
- **Correlated RB** [20]**:** Estimating correlated crosstalk error between qubits participating in separate gates



RB methods for other groups and gates

- **Non-Clifford Dihedral and CNOT-Dihedral RB** [4,6,29]**:**

$$D_m = \langle X, Z_m\rangle$$
$$Z_m=\begin{bmatrix}1&0\\0&e^{2\pi i/m}\end{bmatrix}$$
$$G_m = \langle CX(i,j), X(j), Z_m(j)\rangle$$

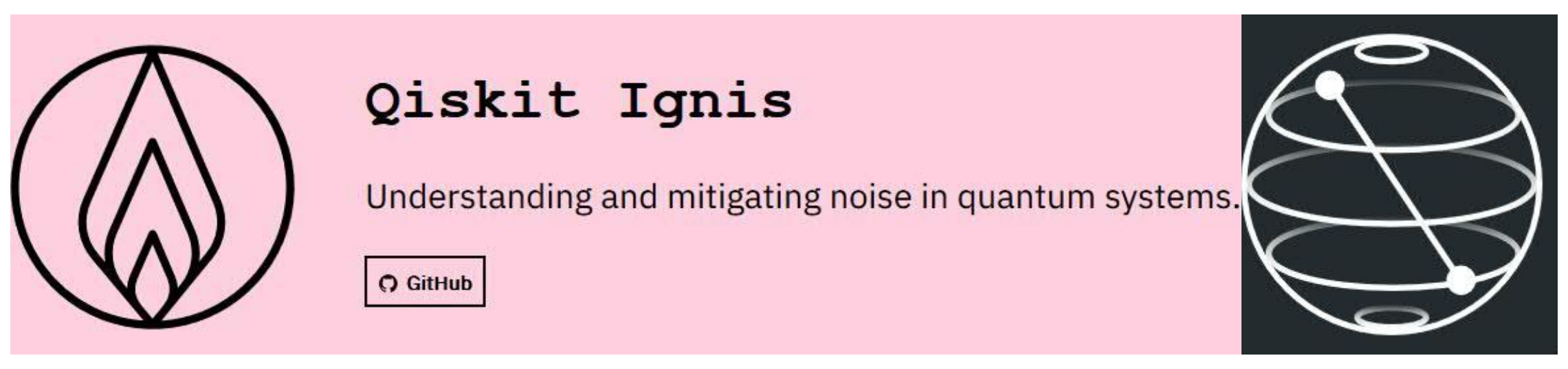


  - Allows benchmarking of non-Clifford gates: T, CS, CCZ (with David McKay and Sarah Sheldon)
  - Efficient synthesis of CNOT-Dihedral elements [5,9] (with Andrew Cross)
- **Pauli and CNOT-Pauli RB** [3]**:**
The Pauli group on n-qubits is generated by the tensor product of the Pauli matrices

$$I=\begin{pmatrix}1&0\\0&1\end{pmatrix},X=\begin{pmatrix}0&1\\1&0\end{pmatrix},Y=\begin{pmatrix}0&-i\\i&0\end{pmatrix},Z=\begin{pmatrix}1&0\\0&-1\end{pmatrix}$$

  - Subgroups of the Clifford group (student project with Jolea Tourk and Merry Tourk)
  - More efficient synthesis compared to the Clifford group
  - These groups form a *unitary 1-design*
- **Direct RB** [17]**:** Directly benchmarking the native gates of a device

RB in Qiskit Ignis

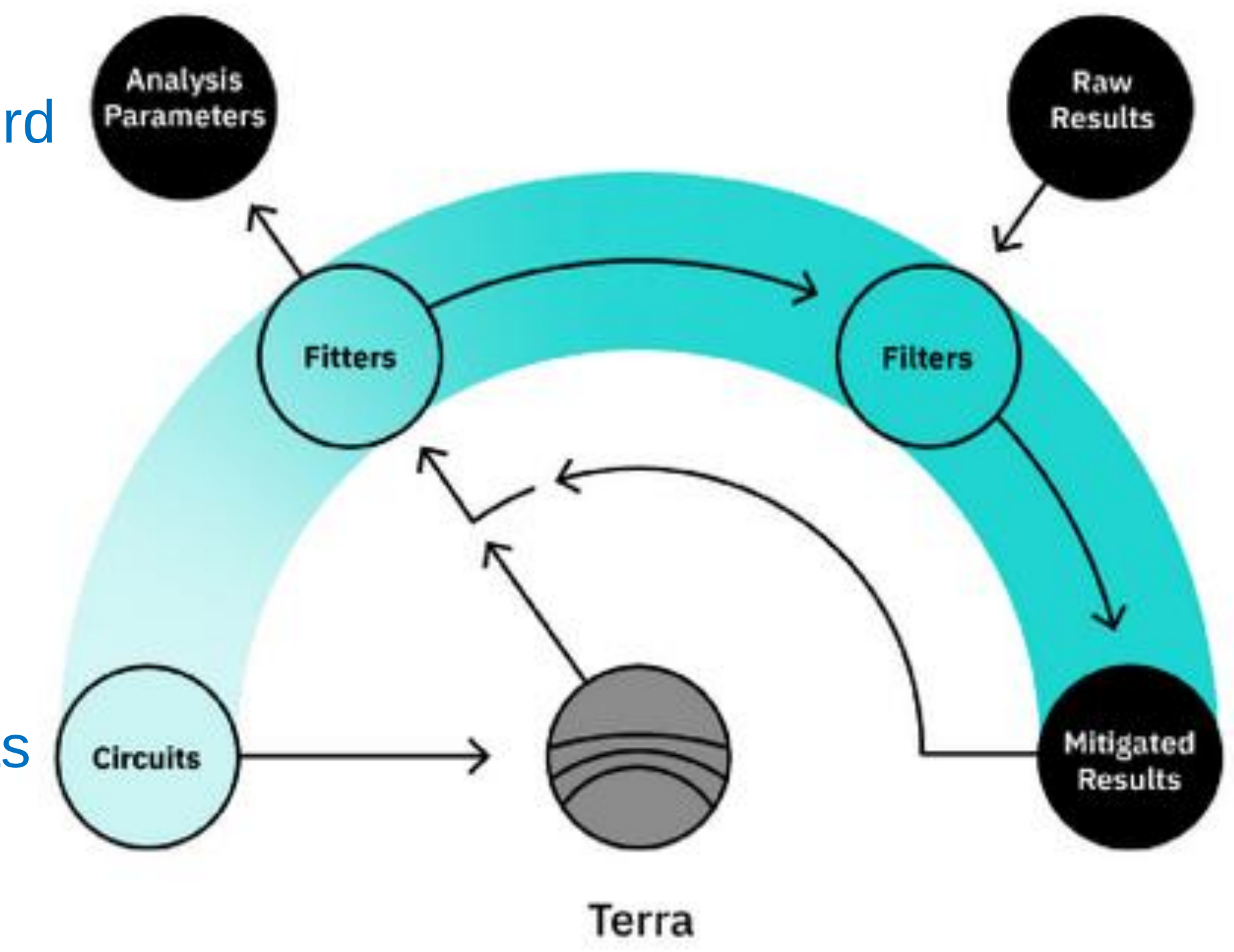


Qiskit Ignis

Understanding and mitigating noise in quantum systems.

Ap^m+B
Error per Clifford

RB circuits



[21,22]

RB code in Qiskit Ignis

```
import qiskit
from qiskit.providers.aer.noise import NoiseModel
from qiskit.providers.aer.noise.errors.standard_errors import depolarizing_error

# Import the RB Functions
from qiskit.ignis.verification.randomized_benchmarking import randomized_benchmarking_seq, RBFitter

# Generate RB circuits (2Q RB)
rb_opts = {}
rb_opts['length_vector'] = [1, 10, 20, 50, 75, 100, 125, 150, 175, 200]
rb_opts['nseeds'] = 5
rb_opts['rb_pattern'] = [[0, 1]]
rb_circs, xdata = randomized_benchmarking_seq(**rb_opts)

# Run on a noisy simulator
noise_model = NoiseModel()
noise_model.add_all_qubit_quantum_error(depolarizing_error(0.002, 1), ['u1', 'u2', 'u3’])
noise_model.add_all_qubit_quantum_error(depolarizing_error(0.002, 2), 'cx’)
backend = qiskit.Aer.get_backend('qasm_simulator’)

# Create the RB fitter
rb_fit = RBFitter(None, xdata, rb_opts['rb_pattern’])
for rb_seed,rb_circ_seed in enumerate(rb_circs):

   job = qiskit.execute(rb_circ_seed, backend=backend,
        basis_gates=['u1','u2','u3','cx'],
        noise_model=noise_model)

   # Add data to the fitter
   rb_fit.add_data(job.result())
   print('After seed %d, EPC %f'%(rb_seed,rb_fit.fit[0]['epc']))
```

References

1. Scott Aaaronson and Daniel Gottesman, *Improved Simulation of Stabilizer Circuits*, Phys. Rev. A 70(052328), 2004. https://arxiv.org/abs/quant-ph/0406196
2. Sergey Bravyi, *Compiling Clifford operators*, *internal note.*
3. Winton G. Brown. and Bryan Eastin, *Randomized benchmarking with restricted gate sets,* Phys. Rev. A 9(062323), 2018, https://arxiv.org/abs/1801.04042
4. Arnaud Carignan-Dugas, Joel Wallman, and Joseph Emerson, *Characterizing Universal Gate Sets via Dihedral Benchmarking*, Phys. Rev. A 92(6), 2015. https://arxiv.org/abs/1508.06312
5. Andrew W. Cross, *internal note*.
6. Andrew W. Cross, Easwar Magesan, Lev S. Bishop, John A. Smolin and Jay M. Gambetta, *Scalable randomised benchmarking of non-Clifford gates*, npj Quantum Inf 2, 16012 (2016). https://arxiv.org/abs/1510.02720
7. A. D. C'orcoles, Jay M. Gambetta, Jerry M. Chow, John A. Smolin, Matthew Ware, J. D. Strand, B. L. T. Plourde, and M. Steffen*, Supplementary material for ''Process verification of two-qubit quantum gates by randomized benchmarking*, Phys. Rev. A 87(030301(R)), 2013. https://arxiv.org/abs/1210.7011
8. Jay M. Gambetta, A. D. C’orcoles, S. T. Merkel, B. R. Johnson, John A. Smolin, Jerry M. Chow, Colm A. Ryan, Chad Rigetti, S. Poletto, Thomas A. Ohki, Mark B. Ketchen, and M. Steffen, *Characterization of addressability by simultaneous randomized benchmarking*, Phys. Rev. Lett. 109(240504) 2012, https://arxiv.org/pdf/1204.6308
9. Shelly Garion and Andrew W. Cross*, On the Structure of the Non-Clifford CNOT-Dihedral Group*, *in preparation.*
10. Daniel Gottesman, *Stabilizer codes and quantum error correction*. PhD dissertation, Caltech, 1997.
11. Vadym Kliuchnikov and Dmitri Maslov*, Optimization of Clifford circuits*, Phys. Rev. A 88(052307), 2013, https://arxiv.org/pdf/1305.0810
12. Robert Koenig and John Smolin, *How to efficiently select an arbitrary Clifford group* element, J. Math. Phys. 55(122202), 2014 https://arxiv.org/abs/1406.2170
13. Easwar Magesan, J. M. Gambetta, and Joseph Emerson, *Robust randomized benchmarking of quantum processes*, Phys. Rev. Lett. 106(180504), 2011. https://arxiv.org/pdf/1009.3639.
14. Easwar Magesan, Jay M. Gambetta, and Joseph Emerson, *Characterizing Quantum Gates via Randomized Benchmarking,* Phys. Rev. A 85(042311), 2012. https://arxiv.org/pdf/1109.6887
15. Easwar Magesan, Jay M. Gambetta, B. R. Johnson, Colm A. Ryan, Jerry M. Chow, Seth T. Merkel, Marcus P. da Silva, George A. Keefe, Mary B. Rothwell, Thomas A. Ohki, Mark B. Ketchen, and M. Steffen, *Efficient measurement of quantum gate error by interleaved randomized benchmarking*, Phys. Rev. Lett. 109(080505), 2012. https://arxiv.org/abs/1203.4550
16. David C. McKay, Stefan Filipp, Antonio Mezzacapo, Easwar Magesan, Jerry M. Chow, and Jay M. Gambetta, *A universal gate for fixed-frequency qubits via a tunable bus*, Phys. Rev. Applied 6(064007), 2016. https://arxiv.org/abs/1604.03076
17. Timothy J. Proctor, Arnaud Carignan-Dugas, Kenneth Rudinger, Erik Nielsen, Robin Blume-Kohout, and Kevin Young, *Direct randomized benchmarking for multi-qubit devices*, Phys. Rev. Lett. 12(030503) 2019, https://arxiv.org/abs/1807.07975
18. Joel Wallman, Chris Granade, Robin Harper, and Steven T. Flammia, *Estimating the Coherence of Noise*, New J. Phys. 17(113020), 2015, https://arxiv.org/abs/1503.07865
19. Christopher J. Wood, Jay Gambetta, *Quantification and Characterization of Leakage Errors*, Phys. Rev. A 97(032306), 2018. https://arxiv.org/abs/1704.03081
20. Christopher J. Wood, David C. d McKay, Andrew W. Cross, Jay Gambetta*, in preparation.*
21. https://qiskit.org/ignis
22. https://github.com/Qiskit/qiskit-ignis
23. https://github.com/Qiskit/qiskit-community-tutorials/blob/master/terra/qis_adv/Clifford_Group.ipynb
24. https://github.com/Qiskit/qiskit-iqx-tutorials/blob/master/qiskit/advanced/ignis/5a_randomized_benchmarking.ipynb
25. https://github.com/Qiskit/qiskit-iqx-tutorials/blob/master/qiskit/advanced/ignis/5b_interleaved_rb.ipynb
26. https://github.com/Qiskit/qiskit-iqx-tutorials/blob/master/qiskit/advanced/ignis/5c_purity_rb.ipynb
27. https://github.com/Qiskit/qiskit-community-tutorials/blob/master/ignis/RB_overview.ipynb
28. https://github.com/Qiskit/qiskit-community-tutorials/blob/master/ignis/RB_advanced.ipynb
29. https://github.com/Qiskit/qiskit-ignis/blob/master/examples/non_clifford_rb.ipynb

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