

MODELLING AND ANALYZING BREAKDOWN PHENOMENA
IN INSULATORS - A STOCHASTIC APPROACH

Research Thesis

Submitted in partial fulfillment of the requirements
for the degree of Doctor of Science

ENMANUEL YASHCHIN

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CONTENTS

	<u>Page No.</u>
Abstract	1
CHAPTER I	3
1. Introduction	3
1.1 The breakdown phenomenon	5
1.1.1 Description	5
1.1.2 Data available	9
1.2 Some physical models of breakdown	10
1.2.1 The model of thermal breakdown	11
1.2.2 Energetic structure of a thin film capacitor	13
1.2.3 Principles of conduction	17
1.2.4 Non-thermal models of breakdown	19
1.3 Statistical questions	27
CHAPTER II	29
2. A Class of Stochastic Models	29
2.1 Pure birth and birth-death models	30
2.2 The quasi-stationary model	32
2.2.1 The charge accumulation process	33
2.2.2 The quasi-stationary distribution	36
2.2.3 Computational aspects	41
2.3 Relevance of Extreme Value Theory	45
2.4 Experimental evidence	48

Contents - Cont'd

	<u>Page No.</u>
CHAPTER III	56
3. Extreme Value Theory - a Survey of Relevant Results	56
3.1 The domain of attraction problem	56
3.2 The joint distribution of extreme order statistics	68
3.3 Applications to statistical inference	71
CHAPTER IV	82
4. Finite Charge Breakdown Models	82
4.1 Domain of attraction	82
4.1.1 Zero initial charge	82
4.1.2 Quasi-stationary case	85
4.2 Regenerative case	86
4.3 Non-regenerative case	87
4.3.1 Zero initial charge	88
4.3.2 Quasi-stationary case	89
CHAPTER V	90
5. Infinite Charge Breakdown Models - Pure Birth Case	90
5.1 Domain of attraction	91
5.1.1 Exact form of the distribution function $P(t)$	92
5.1.2 Quadratic rates	96
5.1.3 General approach	98
5.1.4 An alternative approach	107

Contents - Cont'd

	<u>Page No.</u>
5.2 Normalizing constants	109
5.2.1 Power charge accumulation rates	109
5.2.2 Geometric charge accumulation rates	123
5.2.3 The direct approach in the geometric rates case	137
5.3 Inference for the pure birth model	138
5.3.1 Regenerative case	139
5.3.2 The non-regenerative case	148
CHAPTER VI	150
6. A More General Model of Breakdown, A Tauberian Theorem	150
6.1 The infinite charge model based on transient birth-death process	150
6.2 A Tauberian theorem	159
6.3 Special case	171
CHAPTER VII	174
7. Conclusions	174
7.1 Resume	174
7.2 Open problems	176
Appendix A. Basic properties of theta-functions	179
Appendix B. Euler's summation formula	183
Appendix C. On the saddle point method	186

Contents - Cont'd.

	<u>Page No.</u>
Appendix D. The classification of birth and death processes	191
Appendix E. Choice of the appropriate function $\theta(\gamma, N, \beta)$ when finding the normalizing constants in the case $X_j = a(j+1)^\alpha$	197
Appendix F. Eigenvalues and eigenvectors of the infinitesimal matrix A	199
Appendix G. Transforming the contour of integration and mapping in the integral form of $F(t)$ in the case $k_j = bk^j$	202
Appendix H. On oscillatory behavior of $\phi(q)$ in the case $\lambda_j = b\lambda^j$	204
Appendix I. Justification of the termwise passage to the limit for finding \tilde{k} in the case of power charge accumulation rates.	205a
Appendix 3. Properties of the estimators $\widehat{1/\gamma}$ and $(\widehat{\ln b_N})$	205c
References	206